Research Methodology

Objectives

- To Enable The Students To Know About The Information Needs Of Management
- To Introduce The Concept Of Scientific Research And The Methods Of Conducting Scientific Enquiry And
- To Introduce The Statistical Tools Of Data Analysis

Unit-I

Unit-II

Unit-III

Unit-IV
Unit-V


[Note: Distribution of Questions between Problems and Theory of this paper must be 40:60 i.e., Problem Questions: 40 % & Theory Questions: 60 %]

REFERENCES

UNIT - I

Introduction

Learning objectives:

After reading this lesson, you should be able to understand:

- Meaning, Objectives And Types Of Research
- Qualities Of Researcher
- Significance Of Research
- Research Process
- Research Problem
- Features, Importance, Characteristics, Concepts And Types Of Research Design
- Case Study Research
- Hypothesis And Its Testing
- Sample Survey And Sampling Methods

1.1 Meaning Of Research:

Research in simple terms refers to search for knowledge. It is a scientific and systematic search for information on a particular topic or issue. It is also known as the art of scientific investigation. Several social scientists have defined research in different ways.

In the Encyclopedia of Social Sciences, D. Slesinger and M. Stephenson (1930) defined research as “the manipulation of things, concepts or symbols for the purpose of generalizing to extend, correct or verify knowledge, whether that knowledge aids in the construction of theory or in the practice of an art”.

According to Redman and Mory (1923), research is a “systematized effort to gain new knowledge”. It is an academic activity and therefore the term should be used in a technical sense. According to Clifford Woody (kothari, 1988), research comprises “defining and redefining problems, formulating hypotheses or suggested solutions; collecting, organizing
and evaluating data; making deductions and reaching conclusions; and finally, carefully testing the conclusions to determine whether they fit the formulated hypotheses”.

Thus, research is an original addition to the available knowledge, which contributes to its further advancement. It is an attempt to pursue truth through the methods of study, observation, comparison and experiment. In sum, research is the search for knowledge, using objective and systematic methods to find solution to a problem.

1.1.1 Objectives Of Research:

The objective of research is to find answers to the questions by applying scientific procedures. In other words, the main aim of research is to find out the truth which is hidden and has not yet been discovered. Although every research study has its own specific objectives, the research objectives may be broadly grouped as follows:

1. To gain familiarity with new insights into a phenomenon (i.e., formulative research studies);
2. To accurately portray the characteristics of a particular individual, group, or a situation (i.e., descriptive research studies);
3. To analyse the frequency with which something occurs (i.e., diagnostic research studies); and
4. To examine the hypothesis of a causal relationship between two variables (i.e., hypothesis-testing research studies).

1.1.2 Research Methods Versus Methodology:

Research methods include all those techniques/methods that are adopted for conducting research. Thus, research techniques or methods are the methods that the researchers adopt for conducting the research studies.

on the other hand, research methodology is the way in which research problems are solved systematically. It is a science of studying how research is conducted scientifically. Under it, the researcher acquaints himself/herself with the various steps generally adopted to study a research problem, along with the underlying logic behind them. Hence, it
is not only important for the researcher to know the research techniques/methods, but also the scientific approach called methodology.

1.1.3 Research Approaches:

There are two main approaches to research, namely quantitative approach and qualitative approach. The quantitative approach involves the collection of quantitative data, which are put to rigorous quantitative analysis in a formal and rigid manner. This approach further includes experimental, inferential, and simulation approaches to research. Meanwhile, the qualitative approach uses the method of subjective assessment of opinions, behaviour and attitudes. Research in such a situation is a function of the researcher's impressions and insights. The results generated by this type of research are either in non-quantitative form or in the form which cannot be put to rigorous quantitative analysis. Usually, this approach uses techniques like indepth interviews, focus group interviews, and projective techniques.

1.1.4 Types Of Research:

There are different types of research. The basic ones are as follows.

1. Descriptive Versus Analytical:

Descriptive research consists of surveys and fact-finding enquiries of different types. The main objective of descriptive research is describing the state of affairs as it prevails at the time of study. The term 'ex post facto research' is quite often used for descriptive research studies in social sciences and business research. The most distinguishing feature of this method is that the researcher has no control over the variables here. He/she has to only report what is happening or what has happened. Majority of the ex post facto research projects are used for descriptive studies in which the researcher attempts to examine phenomena, such as the consumers’ preferences, frequency of purchases, shopping, etc. Despite the inability of the researchers to control the variables, ex post facto studies may also comprise attempts by them to discover the causes of the selected problem. The methods of research adopted in conducting descriptive research are survey methods of all kinds, including correlational and comparative methods.
Meanwhile in the Analytical research, the researcher has to use the already available facts or information, and analyse them to make a critical evaluation of the subject.

2. Applied Versus Fundamental:

Research can also be applied or fundamental in nature. An attempt to find a solution to an immediate problem encountered by a firm, an industry, a business organisation, or the society is known as applied research. Researchers engaged in such researches aim at drawing certain conclusions confronting a concrete social or business problem.

On the other hand, fundamental research mainly concerns generalizations and formulation of a theory. In other words, “Gathering knowledge for knowledge’s sake is termed ‘pure’ or ‘basic’ research” (Young in Kothari, 1988). Researches relating to pure mathematics or concerning some natural phenomenon are instances of Fundamental Research. Likewise, studies focusing on human behaviour also fall under the category of fundamental research.

Thus, while the principal objective of applied research is to find a solution to some pressing practical problem, the objective of basic research is to find information with a broad base of application and add to the already existing organized body of scientific knowledge.

3. Quantitative Versus Qualitative:

Quantitative research relates to aspects that can be quantified or can be expressed in terms of quantity. It involves the measurement of quantity or amount. Various available statistical and econometric methods are adopted for analysis in such research. Which includes correlation, regressions and time series analysis etc.

On the other hand, Qualitative research is concerned with qualitative phenomena, or more specifically, the aspects related to or involving quality or kind. For example, an important type of qualitative research is 'Motivation Research', which investigates into the reasons for certain human behaviour. The main aim of this type of research is discovering the underlying motives and desires of human beings by using
in-depth interviews. The other techniques employed in such research are story completion tests, sentence completion tests, word association tests, and other similar projective methods. Qualitative research is particularly significant in the context of behavioural sciences, which aim at discovering the underlying motives of human behaviour. Such research helps to analyse the various factors that motivate human beings to behave in a certain manner, besides contributing to an understanding of what makes individuals like or dislike a particular thing. However, it is worth noting that conducting qualitative research in practice is considerably a difficult task. Hence, while undertaking such research, seeking guidance from experienced expert researchers is important.

4. Conceptual Versus Empirical:

The research related to some abstract idea or theory is known as Conceptual Research. Generally, philosophers and thinkers use it for developing new concepts or for reinterpreting the existing ones. Empirical Research, on the other hand, exclusively relies on the observation or experience with hardly any regard for theory and system. Such research is data based, which often comes up with conclusions that can be verified through experiments or observation. Empirical research is also known as experimental type of research, in which it is important to first collect the facts and their sources, and actively take steps to stimulate the production of desired information. In this type of research, the researcher firstformulates a working hypothesis, and then gathers sufficient facts to prove or disprove the stated hypothesis. He/she formulates the experimental design, which according to him/her would manipulate the variables, so as to obtain the desired information. This type of research is thus characterized by the researcher’s control over the variables under study. In simple term, empirical research is most appropriate when an attempt is made to prove that certain variables influence the other variables in some way. Therefore, the results obtained by using the experimental or empirical studies are considered to be the most powerful evidences for a given hypothesis.

5. Other Types Of Research:

The remaining types of research are variations of one or more of the afore-mentioned type of research. They vary in terms of the purpose of research, or the time required to complete it, or may be based on some
other similar factor. On the basis of time, research may either be in the nature of one-time or longitudinal time series research. While the research is restricted to a single time-period in the former case, it is conducted over several time-periods in the latter case. Depending upon the environment in which the research is to be conducted, it can also be laboratory research or field-setting research, or simulation research, besides being diagnostic or clinical in nature. Under such research, in-depth approaches or case study method may be employed to analyse the basic causal relations. These studies usually undertake a detailed in-depth analysis of the causes of certain events of interest, and use very small samples and sharp data collection methods. The research may also be explanatory in nature. Formalized research studies consist of substantial structure and specific hypotheses to be verified. As regards to historical research, sources like historical documents, remains, etc. Are utilized to study past events or ideas. It also includes philosophy of persons and groups of the past or any remote point of time.

Research has also been classified into decision-oriented and conclusion-oriented categories. The decision-oriented research is always carried out as per the need of a decision maker and hence, the researcher has no freedom to conduct the research according to his/her own desires. On the other hand, in the case of Conclusion-oriented research, the researcher is free to choose the problem, redesign the enquiry as it progresses and even change conceptualization as he/she wishes to. Operations research is a kind of decision-oriented research, where in scientific method is used in providing the departments, a quantitative basis for decision-making with respect to the activities under their purview.

1.1.5 Importance Of Knowing How To Conduct Research:

The importance of knowing how to conduct research are listed below:

i. The knowledge of research methodology provides training to new researchers and enables them to do research properly. It helps them to develop disciplined thinking or a 'bent of mind' to objectively observe the field;

ii. The knowledge of doing research inculcates the ability to evaluate and utilize the research findings with confidence;
iii. The knowledge of research methodology equips the researcher with the tools that help him/her to make the observations objectively; and

iv. The knowledge of methodology helps the research consumers to evaluate research and make rational decisions.

### 1.1.6 Qualities Of A Researcher:

It is important for a researcher to possess certain qualities to conduct research. First and foremost, he being a scientist should be firmly committed to the ‘articles of faith’ of the scientific methods of research. This implies that a researcher should be a social science person in the truest sense. Sir Michael Foster cited by (Wilkinson and Bhandarkar, 1979) identified a few distinct qualities of a scientist. According to him, a true research scientist should possess the following qualities:

1. First of all, the nature of a researcher must be of the temperament that vibrates in unison with the theme which he is searching. Hence, the seeker of knowledge must be truthful with truthfulness of nature, which is much more important, much more exacting than what is sometimes known as truthfulness. The truthfulness relates to the desire for accuracy of observation and precision of statement. Ensuring facts is the principle rule of science, which is not an easy matter. The difficulty may arise due to untrained eye, which fails to see anything beyond what it has the power of seeing and sometimes even less than that. This may also be due to the lack of discipline in the method of science. An unscientific individual often remains satisfied with the expressions like approximately, almost, or nearly, which is never what nature is. A real research cannot see two things which differ, however minutely, as the same.

2. A researcher must possess an alert mind. Nature is constantly changing and revealing itself through various ways. A scientific researcher must be keen and watchful to notice such changes, no matter how small or insignificant they may appear. Such receptivity has to be cultivated slowly and patiently over time by the researcher through practice. An individual who is ignorant or not alert and receptive during his research will not make a good researcher. He will fail as a good researcher if he has no keen eyes or mind to observe the unusual changes behind the routine. Research
demands a systematic immersion into the subject matter by the researcher
grasp even the slightest hint that may culminate into significant research
problems. In this context, Cohen and Negal cited by (Selltiz et al, 1965;
Wilkinson and Bhandarkar, 1979) state that “the ability to perceive in some
brute experience the occasion of a problem is not a common talent among
men... it is a mark of scientific genius to be sensitive to difficulties where
less gifted people pass by untroubled by doubt”.

(3) Scientific enquiry is pre-eminently an intellectual effort. It
requires the moral quality of courage, which reflects the courage of a
steadfast endurance. The process of conducting research is not an easy
task. There are occasions when a research scientist might feel defeated or
completely lost. This is the stage when a researcher would need immense
courage and the sense of conviction. The researcher must learn the art of
enduring intellectual hardships. In the words of Darwin, “It’s dogged that
does it”.

In order to cultivate the afore-mentioned three qualities of a
researcher, a fourth one may be added. This is the quality of making
statements cautiously. According to Huxley, the assertion that outstrips the
evidence is not only a blunder but a crime (Thompson, 1975). A researcher
should cultivate the habit of reserving judgment when the required data
are insufficient.

1.1.7 Significance Of Research:

According to a famous Hudson Maxim, “All progress is born of
inquiry. Doubt is often better than overconfidence, for it leads to inquiry,
and inquiry leads to invention”. It brings out the significance of research,
increased amount of which makes the progress possible. Research
encourages scientific and inductive thinking, besides promoting the
development of logical habits of thinking and organisation. The role of
research in applied economics in the context of an economy or business
is greatly increasing in modern times. The increasingly complex nature
of government and business has raised the use of research in solving
operational problems. Research assumes significant role in the formulation
of economic policy for both, the government and business. It provides
the basis for almost all government policies of an economic system.
Government budget formulation, for example, depends particularly on the
analysis of needs and desires of people, and the availability of revenues, which requires research. Research helps to formulate alternative policies, in addition to examining the consequences of these alternatives. Thus, research also facilitates the decision-making of policy-makers, although in itself is not a part of research. In the process, research also helps in the proper allocation of a country’s scarce resources.

Research is also necessary for collecting information on the social and economic structure of an economy to understand the process of change occurring in the country. Collection of statistical information, though not a routine task, involves various research problems. Therefore, large staff of research technicians or experts are engaged by the government these days to undertake this work. Thus, research as a tool of government economic policy formulation involves three distinct stages of operation: (i) investigation of economic structure through continual compilation of facts; (ii) diagnosis of events that are taking place and analysis of the forces underlying them; and (iii) the prognosis i.e., the prediction of future developments (Wilkinson and Bhandarkar, 1979).

Research also assumes significance in solving various operational and planning problems associated with business and industry. In several ways, operations research, market research and motivational research are vital and their results assist in taking business decisions. Market research refers to the investigation of the structure and development of a market for the formulation of efficient policies relating to purchases, production and sales. Operational research relates to the application of logical, mathematical, and analytical techniques to find solution to business problems, such as cost minimization or profit maximization, or the optimization problems. Motivational research helps to determine why people behave in the manner they do with respect to market characteristics. More specifically, it is concerned with the analysis of the motivations underlying consumer behaviour. All these researches are very useful for business and industry, and are responsible for business decision-making.

Research is equally important to social scientists for analyzing the social relationships and seeking explanations to various social problems. It gives intellectual satisfaction of knowing things for the sake of knowledge. It also possesses the practical utility for the social scientist to gain knowledge so as to be able to do something better or in a more
efficient manner. The research in social sciences is concerned with both knowledge for its own sake, and knowledge for what it can contribute to solve practical problems.

1.2 Research Process:

Research process consists of a series of steps or actions required for effectively conducting research. The following are the steps that provide useful procedural guidelines regarding the conduct of research:

(1) Formulating the research problem;
(2) Extensive literature survey;
(3) Developing hypothesis;
(4) Preparing the research design;
(5) Determining sample design;
(6) Collecting data;
(7) Execution of the project;
(8) Analysis of data;
(9) Hypothesis testing;
(10) Generalization and interpretation, and
(11) Preparation of the report or presentation of the results.

In other words, it involves the formal write-up of conclusions.

1.3 Research Problem:

The first and foremost stage in the research process is to select and properly define the research problem. A researcher should first identify a problem and formulate it, so as to make it amenable or susceptible to research. In general, a research problem refers to an unanswered question that a researcher might encounter in the context of either a theoretical or practical situation, which he/she would like to answer or find a solution to. A research problem is generally said to exist if the following conditions emerge (Kothari, 1988):

i. There should be an individual or an organisation, say X, to whom the Problem can be attributed. The individual or the organization is situated in an environment Y, which is governed by certain uncontrolled variables Z;
ii. There should be at least two courses of action to be pursued, say A1 and A2. These courses of action are defined by one or more values of the controlled variables. For example, the number of items purchased at a specified time is said to be one course of action.

iii. There should be at least two alternative possible outcomes of the said courses of action, say B1 and B2. Of them, one alternative should be preferable to the other. That is, at least one outcome should be what the researcher wants, which becomes an objective.

iv. The courses of possible action available must offer a chance to the researcher to achieve the objective, but not the equal chance. Therefore, if P(Bj / X, A, Y) represents the probability of the occurrence of an outcome Bj when X selects Aj in Y, then P(B1 / X, A1, Y) ≠ P(B1 / X, A2, Y). Putting it in simple words, it means that the choices must not have equal efficiencies for the desired outcome.

Above all these conditions, the individual or organisation may be said to have arrived at the research problem only if X does not know what course of action to be taken is the best. In other words, X should have a doubt about the solution. Thus, an individual or a group of persons can be said to have a problem if they have more than one desired outcome. They should have two or more alternative courses of action, which have some but not equal efficiency. This is required for probing the desired objectives, such that they have doubts about the best course of action to be taken. Thus, the components of a research problem may be summarised as:

i. There should be an individual or a group who have some difficulty or problem.

ii. There should be some objective(s) to be pursued. A person or an organization who wants nothing cannot have a problem.

iii. There should be alternative ways of pursuing the objective the researcher wants to pursue. This implies that there should be more than one alternative means available to the researcher. This is because if the researcher has no choice of alternative means, he/she would not have a problem.
iv. There should be some doubt in the mind of the researcher about the choice of alternative means. This implies that research should answer the question relating to the relative efficiency or suitability of the possible alternatives.

v. There should be a context to which the difficulty relates.

Thus, identification of a research problem is the pre-condition to conducting research. A research problem is said to be the one which requires a researcher to find the best available solution to the given problem. That is, the researcher needs to find out the best course of action through which the research objective may be achieved optimally in the context of a given situation. Several factors may contribute to making the problem complicated. For example, the environment may alter, thus affecting the efficiencies of the alternative courses of action taken or the quality of the outcomes. The number of alternative courses of action might be very large and the individual not involved in making the decision may be affected by the change in environment and may react to it favorably or unfavorably. Other similar factors are also likely to cause such changes in the context of research, all of which may be considered from the point of view of a research problem.

1.4 Research Design:

The most important step after defining the research problem is preparing the design of the research project, which is popularly known as the ‘research design’. A research design helps to decide upon issues like what, when, where, how much, by what means etc. With regard to an enquiry or a research study. A research design is the arrangement of conditions for collection and analysis of data in a manner that aims to combine relevance to the research purpose with economy in procedure. Infact, research design is the conceptual structure within which research is conducted; it constitutes the blueprint for the collection, measurement and analysis of data (Sellitz et al, 1962). Thus, research design provides an outline of what the researcher is going to do in terms of framing the hypothesis, its operational implications and the final data analysis. Specifically, the research design highlights decisions which include:
1. The nature of the study
2. The purpose of the study
3. The location where the study would be conducted
4. The nature of data required
5. From where the required data can be collected
6. What time period the study would cover
7. The type of sample design that would be used
8. The techniques of data collection that would be used
9. The methods of data analysis that would be adopted and
10. The manner in which the report would be prepared

In view of the stated research design decisions, the overall research design may be divided into the following (Kothari 1988):

a. The sampling design that deals with the method of selecting items to be observed for the selected study;
b. The observational design that relates to the conditions under which the observations are to be made;
c. The statistical design that concerns with the question of how many items are to be observed, and how the information and data gathered are to be analysed; and
d. The operational design that deals with the techniques by which the procedures specified in the sampling, statistical and observational designs can be carried out.

1.4.1 **Features Of Research Design:**

The important features of Research Design may be outlined as follows:

i. It constitutes a plan that identifies the types and sources of information required for the research problem;
ii. It constitutes a strategy that specifies the methods of data collection and analysis which would be adopted; and
iii. It also specifies the time period of research and monetary budget involved in conducting the study, which comprise the two major constraints of undertaking any research
1.4.2 Concepts Relating To Research Design:

Some of the important concepts relating to Research Design are discussed below:

1. Dependent And Independent Variables:

   A magnitude that varies is known as a variable. The concept may assume different quantitative values like height, weight, income etc. Qualitative variables are not quantifiable in the strictest sense of the term. However, the qualitative phenomena may also be quantified in terms of the presence or absence of the attribute(s) considered. The phenomena that assume different values quantitatively even in decimal points are known as 'continuous variables'. But all variables need not be continuous. Values that can be expressed only in integer values are called 'non-continuous variables'. In statistical terms, they are also known as 'discrete variables'. For example, age is a continuous variable, whereas the number of children is a non-continuous variable. When changes in one variable depend upon the changes in other variable or variables, it is known as a dependent or endogenous variable, and the variables that cause the changes in the dependent variable are known as the independent or explanatory or exogenous variables. For example, if demand depends upon price, then demand is a dependent variable, while price is the independent variable. And, if more variables determine demand, like income and price of the substitute commodity, then demand also depends upon them in addition to the price of original commodity. In other words, demand is a dependent variable which is determined by the independent variables like price of the original commodity, income and price of substitutes.

2. Extraneous Variables:

   The independent variables which are not directly related to the purpose of the study but affect the dependent variables, are known as extraneous variables. For instance, assume that a researcher wants to test the hypothesis that there is a relationship between children's school performance and their self-confidence, in which case the latter is an independent variable and the former, a dependent variable. In this context, intelligence may also influence the school performance. However, since it is not directly related to the purpose of the study undertaken by the
researcher, it would be known as an extraneous variable. The influence caused by the extraneous variable(s) on the dependent variable is technically called the ‘experimental error’. Therefore, a research study should always be framed in such a manner that the influence of extraneous variables on the dependent variable/s is completely controlled, and the influence of independent variable/s is clearly evident.

3. Control:

One of the most important features of a good research design is to minimize the effect of extraneous variable(s). Technically, the term ‘control’ is used when a researcher designs the study in such a manner that it minimizes the effects of extraneous variables. The term ‘control’ is used in experimental research to reflect the restrain in experimental conditions.

4. Confounded Relationship:

The relationship between the dependent and independent variables is said to be confounded by an extraneous variable, when the dependent variable is not free from its effects.

5. Research Hypothesis:

When a prediction or a hypothesized relationship is tested by adopting scientific methods, it is known as research hypothesis. The research hypothesis is a predictive statement which relates to a dependent variable and an independent variable. Generally, a research hypothesis must consist of at least one dependent variable and one independent variable. Whereas, the relationships that are assumed but not to be tested are predictive statements that are not to be objectively verified, thus are not classified as research hypotheses.

6. Experimental and Non-experimental Hypothesis Testing Research:

When the objective of a research is to test a research hypothesis, it is known as hypothesis-testing research. Such research may be in the nature of experimental design or non-experimental design. The research in which the independent variable is manipulated is known as ‘experimental hypothesis-testing research’, whereas the research in which the independent
variable is not manipulated is termed as ‘non-experimental hypothesis-testing research’. For example, assume that a researcher wants to examine whether family income influences the school attendance of a group of students, by calculating the coefficient of correlation between the two variables. Such an example is known as a non-experimental hypothesis-testing research, because the independent variable - family income is not manipulated here. Again assume that the researcher randomly selects 150 students from a group of students who pay their school fees regularly and then classifies them into two sub-groups by randomly including 75 in Group A, whose parents have regular earning, and 75 in Group B, whose parents do not have regular earning. Assume that at the end of the study, the researcher conducts a test on each group in order to examine the effects of regular earnings of the parents on the school attendance of the student. Such a study is an example of experimental hypothesis-testing research, because in this particular study the independent variable regular earnings of the parents have been manipulated.

7. Experimental And Control Groups:

When a group is exposed to usual conditions in an experimental hypothesis-testing research, it is known as ‘control group’. On the other hand, when the group is exposed to certain new or special condition, it is known as an ‘experimental group’. In the afore-mentioned example, Group A can be called as control group and Group B as experimental group. If both the groups, A and B are exposed to some special feature, then both the groups may be called as ‘experimental groups’. A research design may include only the experimental group or both the experimental and control groups together.

8. Treatments:

Treatments refer to the different conditions to which the experimental and control groups are subject to. In the example considered, the two treatments are the parents with regular earnings and those with no regular earnings. Likewise, if a research study attempts to examine through an experiment the comparative effect of three different types of fertilizers on the yield of rice crop, then the three types of fertilizers would be treated as the three treatments.
9. Experiment:

Experiment refers to the process of verifying the truth of a statistical hypothesis relating to a given research problem. For instance, an experiment may be conducted to examine the yield of a certain new variety of rice crop developed. Further, Experiments may be categorized into two types, namely, ‘absolute experiment’ and ‘comparative experiment’. If a researcher wishes to determine the impact of a chemical fertilizer on the yield of a particular variety of rice crop, then it is known as absolute experiment. Meanwhile, if the researcher wishes to determine the impact of chemical fertilizer as compared to the impact of bio-fertilizer, then the experiment is known as a comparative experiment.

10. Experimental Unit(s):

Experimental units refer to the pre-determined plots, characteristics or the blocks, to which different treatments are applied. It is worth mentioning here that such experimental units must be selected with great caution.

1.4.3 Types Of Research Design:

There are different types of research designs. They may be broadly categorized as:

(1) Exploratory Research Design;
(2) Descriptive and Diagnostic Research Design; and
(3) Hypothesis-Testing Research Design.

1. Exploratory Research Design:

The Exploratory Research Design is known as formulative research design. The main objective of using such a research design is to formulate a research problem for an in-depth or more precise investigation, or for developing a working hypothesis from an operational aspect. The major purpose of such studies is the discovery of ideas and insights. Therefore, such a research design suitable for such a study should be flexible enough to provide opportunity for considering different dimensions of the problem under study. The in-built flexibility in research design is required as the
initial research problem would be transformed into a more precise one in the exploratory study, which in turn may necessitate changes in the research procedure for collecting relevant data. Usually, the following three methods are considered in the context of a research design for such studies. They are (a) a survey of related literature; (b) experience survey; and (c) analysis of ‘insight-stimulating’ instances.

2. Descriptive And Diagnostic Research Design:

A Descriptive Research Design is concerned with describing the characteristics of a particular individual or a group. Meanwhile, a diagnostic research design determines the frequency with which a variable occurs or its relationship with another variable. In other words, the study analyzing whether a certain variable is associated with another comprises a diagnostic research study. On the other hand, a study that is concerned with specific predictions or with the narration of facts and characteristics related to an individual, group or situation, are instances of descriptive research studies. Generally, most of the social research design falls under this category. As a research design, both the descriptive and diagnostic studies share common requirements, hence they are grouped together. However, the procedure to be used and the research design need to planned carefully. The research design must also make appropriate provision for protection against bias and thus maximize reliability, with due regard to the completion of the research study in an economical manner. The research design in such studies should be rigid and not flexible. Besides, it must also focus attention on the following:

a) Formulation of the objectives of the study,
b) Proper designing of the methods of data collection,
c) Sample selection,
d) Data collection,
e) Processing and analysis of the collected data, and
f) Reporting the findings.
3. **Hypothesis-Testing Research Design:**

Hypothesis-Testing Research Designs are those in which the researcher tests the hypothesis of causal relationship between two or more variables. These studies require procedures that would not only decrease bias and enhance reliability, but also facilitate deriving inferences about the causality. Generally, experiments satisfy such requirements. Hence, when research design is discussed in such studies, it often refers to the design of experiments.

**1.4.4 Importance Of Research Design:**

The need for a research design arises out of the fact that it facilitates the smooth conduct of the various stages of research. It contributes to making research as efficient as possible, thus yielding the maximum information with minimum effort, time and expenditure. A research design helps to plan in advance, the methods to be employed for collecting the relevant data and the techniques to be adopted for their analysis. This would help in pursuing the objectives of the research in the best possible manner, provided the available staff, time and money are given. Hence, the research design should be prepared with utmost care, so as to avoid any error that may disturb the entire project. Thus, research design plays a crucial role in attaining the reliability of the results obtained, which forms the strong foundation of the entire process of the research work.

Despite its significance, the purpose of a well-planned design is not realized at times. This is because it is not given the importance that it deserves. As a consequence, many researchers are not able to achieve the purpose for which the research designs are formulated, due to which they end up arriving at misleading conclusions. Therefore, faulty designing of the research project tends to render the research exercise meaningless. This makes it imperative that an efficient and suitable research design must be planned before commencing the process of research. The research design helps the researcher to organize his/her ideas in a proper form, which in turn facilitates him/her to identify the inadequacies and faults in them. The research design is also discussed with other experts for their comments and critical evaluation, without which it would be difficult for any critic to provide a comprehensive review and comments on the proposed study.
1.4.5 Characteristics Of A Good Research Design:

A good research design often possesses the qualities of being flexible, suitable, efficient, economical and so on. Generally, a research design which minimizes bias and maximizes the reliability of the data collected and analysed is considered a good design (Kothari 1988). A research design which does not allow even the smallest experimental error is said to be the best design for investigation. Further, a research design that yields maximum information and provides an opportunity of viewing the various dimensions of a research problem is considered to be the most appropriate and efficient design. Thus, the question of a good design relates to the purpose or objective and nature of the research problem studied. While a research design may be good, it may not be equally suitable to all studies. In other words, it may be lacking in one aspect or the other in the case of some other research problems. Therefore, no single research design can be applied to all types of research problems.

A research design suitable for a specific research problem would usually involve the following considerations:

i. The methods of gathering the information;
ii. The skills and availability of the researcher and his/her staff, if any;
iii. The objectives of the research problem being studied;
iv. The nature of the research problem being studied; and
v. The available monetary support and duration of time for the research work.

1.5 Case Study Research:

The method of exploring and analyzing the life or functioning of a social or economic unit, such as a person, a family, a community, an institution, a firm or an industry is called case study method. The objective of case study method is to examine the factors that cause the behavioural patterns of a given unit and its relationship with the environment. The data for a study are always gathered with the purpose of tracing the natural history of a social or economic unit, and its relationship with the social or economic factors, besides the forces involved in its environment. Thus, a researcher conducting a study using the case study method attempts to
understand the complexity of factors that are operative within a social or economic unit as an integrated totality. Burgess (Kothari, 1988) described the special significance of the case study in understanding the complex behaviour and situations in specific detail. In the context of social research, he called such data as social microscope.

1.5.1 Criteria For Evaluating Adequacy Of Case Study:

John Dollard (Dollard, 1935) specified seven criteria for evaluating the adequacy of a case or life history in the context of social research. They are:

i. The subject being studied must be viewed as a specimen in a cultural set up. That is, the case selected from its total context for the purpose of study should be considered a member of the particular cultural group or community. The scrutiny of the life history of the individual must be carried out with a view to identify the community values, standards and shared ways of life.

ii. The organic motors of action should be socially relevant. This is to say that the action of the individual cases should be viewed as a series of reactions to social stimuli or situations. To put in simple words, the social meaning of behaviour should be taken into consideration.

iii. The crucial role of the family-group in transmitting the culture should be recognized. This means, as an individual is the member of a family, the role of the family in shaping his/her behaviour should never be ignored.

iv. The specific method of conversion of organic material into social behaviour should be clearly demonstrated. For instance, case-histories that discuss in detail how basically a biological organism, that is man, gradually transforms into a social person are particularly important.

v. The constant transformation of character of experience from childhood to adulthood should be emphasized. That is, the life-history should portray the inter-relationship between the individual's various experiences during his/her life span. Such a study provides a comprehensive understanding of an individual’s life as a continuum.
vi. The ‘social situation’ that contributed to the individual’s gradual transformation should carefully and continuously be specified as a factor. One of the crucial criteria for life-history is that an individual’s life should be depicted as evolving itself in the context of a specific social situation and partially caused by it.

vii. The life-history details themselves should be organized according to some conceptual framework, which in turn would facilitate their generalizations at higher levels.

These criteria discussed by Dollard emphasize the specific link of co-ordinated, related, continuous and configured experience in a cultural pattern that motivated the social and personal behaviour. Although, the criteria indicated by Dollard are principally perfect, some of them are difficult to put to practice.

Dollard (1935) attempted to express the diverse events depicted in the life-histories of persons during the course of repeated interviews by utilizing psycho-analytical techniques in a given situational context. His criteria of life-history originated directly from this experience. While the life-histories possess independent significance as research documents, the interviews recorded by the investigators can afford, as Dollard observed, “rich insights into the nature of the social situations experienced by them”.

It is a well-known fact that an individual’s life is very complex. Till date there is hardly any technique that can establish some kind of uniformity, and as a result ensure the cumulative of case-history materials by isolating the complex totality of a human life. Nevertheless, although case history data are difficult to put to rigorous analysis, a skilful handling and interpretation of such data could help in developing insights into cultural conflicts and problems arising out of cultural-change.

Gordon Allport in (Kothari 1988) has recommended the following aspects so as to broaden the perspective of case-study data:

i. If the life-history is written in first person, it should be as comprehensive and coherent as possible.

ii. Life-histories must be written for knowledgeable persons. That is, if the enquiry of study is sociological in nature, the researcher should
write it on the assumption that it would be read largely by sociologists only.

iii. It would be advisable to supplement case study data by observational, statistical and historical data, as they provide standards for assessing the reliability and consistency of the case study materials. Further, such data offer a basis for generalizations.

iv. Efforts must be made to verify the reliability of life-history data by examining the internal consistency of the collected material, and by repeating the interviews with the concerned person. Besides this, personal interviews with the persons who are well-acquainted with him/her, belonging to his/her own group should be conducted.

v. A judicious combination of different techniques for data-collection is crucial for collecting data that are culturally meaningful and scientifically significant.

vi. Life-histories or case-histories may be considered as an adequate basis for generalization to the extent that they are typical or representative of a certain group.

vii. The researcher engaged in the collection of case study data should never ignore the unique or typical cases. He/she should include them as exceptional cases.

Case histories are filled with valuable information of a personal or private nature. Such information not only helps the researcher to portray the personality of the individual, but also the social background that contributed to it. Besides, it also helps in the formulation of relevant hypotheses. In general, although Blummer (in Wilkinson and Bhandarkar, 1979) was critical of documentary material, he gave due credit to case histories by acknowledging the fact that the personal documents offer an opportunity to the researcher to develop his/her spirit of enquiry. The analysis of a particular subject would be more effective if the researcher acquires close acquaintance with it through personal documents. However, Blummer also acknowledges the limitations of the personal documents. According to him, such documents do not entirely fulfill the criteria of adequacy, reliability, and representativeness. Despite these shortcomings, avoiding their use in any scientific study of personal life would be wrong, as these documents become necessary and significant for both theory-building and practice.
In spite of these formidable limitations, case study data are used by anthropologists, sociologists, economists and industrial psychiatrists. Gordon Allport (Kothari, 1988) strongly recommends the use of case study data for in-depth analysis of a subject. For, it is one's acquaintance with an individual that instills a desire to know his/her nature and understand them. The first stage involves understanding the individual and all the complexity of his/her nature. Any haste in analyzing and classifying the individual would create the risk of reducing his/her emotional world into artificial bits. As a consequence, the important emotional organizations, anchorages and natural identifications characterizing the personal life of the individual might not yield adequate representation. Hence, the researcher should understand the life of the subject. Therefore, the totality of life-processes reflected in the well-ordered life-history documents become invaluable source of stimulating insights. Such life-history documents provide the basis for comparisons that contribute to statistical generalizations and help to draw inferences regarding the uniformities in human behaviour, which are of great value. Even if some personal documents do not provide ordered data about personal lives of people, which is the basis of psychological science, they should not be ignored. This is because the final aim of science is to understand, control and make predictions about human life. Once they are satisfied, the theoretical and practical importance of personal documents must be recognized as significant. Thus, a case study may be considered as the beginning and the final destination of abstract knowledge.

1.6 Hypothesis:

“Hypothesis may be defined as a proposition or a set of propositions set forth as an explanation for the occurrence of some specified group of phenomena either asserted merely as a provisional conjecture to guide some investigation in the light of established facts” (Kothari, 1988). A research hypothesis is quite often a predictive statement, which is capable of being tested using scientific methods that involve an independent and some dependent variables. For instance, the following statements may be considered:

i. “Students who take tuitions perform better than the others who do not receive tuitions” or,

ii. “The female students perform as well as the male students”.

26
These two statements are hypotheses that can be objectively verified and tested. Thus, they indicate that a hypothesis states what one is looking for. Besides, it is a proposition that can be put to test in order to examine its validity.

1.6.1 Characteristics Of Hypothesis:

A hypothesis should have the following characteristic features:-

i. A hypothesis must be precise and clear. If it is not precise and clear, then the inferences drawn on its basis would not be reliable.

ii. A hypothesis must be capable of being put to test. Quite often, the research programmes fail owing to its incapability of being subject to testing for validity. Therefore, some prior study may be conducted by the researcher in order to make a hypothesis testable. A hypothesis “is tested if other deductions can be made from it, which in turn can be confirmed or disproved by observation” (Kothari, 1988).

iii. A hypothesis must state relationship between two variables, in the case of relational hypotheses.

iv. A hypothesis must be specific and limited in scope. This is because a simpler hypothesis generally would be easier to test for the researcher. And therefore, he/she must formulate such hypotheses.

v. As far as possible, a hypothesis must be stated in the simplest language, so as to make it understood by all concerned. However, it should be noted that simplicity of a hypothesis is not related to its significance.

vi. A hypothesis must be consistent and derived from the most known facts. In other words, it should be consistent with a substantial body of established facts. That is, it must be in the form of a statement which is most likely to occur.

vii. A hypothesis must be amenable to testing within a stipulated or reasonable period of time. No matter how excellent a hypothesis, a researcher should not use it if it cannot be tested within a given period of time, as no one can afford to spend a life-time on collecting data to test it.
A hypothesis should state the facts that give rise to the necessity of looking for an explanation. This is to say that by using the hypothesis, and other known and accepted generalizations, a researcher must be able to derive the original problem condition. Therefore, a hypothesis should explain what it actually wants to explain, and for this it should also have an empirical reference.

1.6.2 Concepts Relating To Testing Of Hypotheses:

Testing of hypotheses requires a researcher to be familiar with various concepts concerned with it such as:

1) Null Hypothesis And Alternative Hypothesis:

In the context of statistical analysis, hypotheses are of two types viz., null hypothesis and alternative hypothesis. When two methods A and B are compared on their relative superiority, and it is assumed that both the methods are equally good, then such a statement is called as the null hypothesis. On the other hand, if method A is considered relatively superior to method B, or vice-versa, then such a statement is known as an alternative hypothesis. The null hypothesis is expressed as $H_0$, while the alternative hypothesis is expressed as $H_a$. For example, if a researcher wants to test the hypothesis that the population mean ($\mu$) is equal to the hypothesized mean ($H_0$) = 100, then the null hypothesis should be stated as the population mean is equal to the hypothesized mean 100. Symbolically it may be written as:

$$H_0: \mu = \mu_0 = 100$$

If sample results do not support this null hypothesis, then it should be concluded that something else is true. The conclusion of rejecting the null hypothesis is called as alternative hypothesis $H_1$. To put it in simple words, the set of alternatives to the null hypothesis is termed as the alternative hypothesis. If $H_0$ is accepted, then it implies that $H_a$ is being rejected. On the other hand, if $H_0$ is rejected, it means that $H_a$ is being accepted. For $H_0: \mu = \mu_0 = 100$, the following three possible alternative hypotheses may be considered:
Before the sample is drawn, the researcher has to state the null hypothesis and the alternative hypothesis. While formulating the null hypothesis, the following aspects need to be considered:

A. Alternative hypothesis is usually the one which a researcher wishes to prove, whereas the null hypothesis is the one which he/she wishes to disprove. Thus, a null hypothesis is usually the one which a researcher tries to reject, while an alternative hypothesis is the one that represents all other possibilities.

B. The rejection of a hypothesis when it is actually true involves great risk, as it indicates that it is a null hypothesis because then the probability of rejecting it when it is true is \( \alpha \) (i.e., the level of significance) which is chosen very small.

C. Null hypothesis should always be specific hypothesis i.e., it should not state about or approximately a certain value.

2) **The Level Of Significance:**

In the context of hypothesis testing, the level of significance is a very important concept. It is a certain percentage that should be chosen with great care, reason and insight. If for instance, the significance level is taken at 5 per cent, then it means that \( H_0 \) would be rejected when the sampling result has a less than 0.05 probability of occurrence when \( H_0 \) is true. In other words, the five per cent level of significance implies that the researcher is willing to take a risk of five per cent of rejecting the null hypothesis, when \( (H_0) \) is actually true. In sum, the significance level reflects the maximum value of the probability of rejecting \( H_0 \) when it is actually true, and which is usually determined prior to testing the hypothesis.
3) **Test Of Hypothesis Or Decision Rule:**

Suppose the given hypothesis is \( H_0 \) and the alternative hypothesis \( H_1 \), then the researcher has to make a rule known as the decision rule. According to the decision rule, the researcher accepts or rejects \( H_0 \). For example, if the \( H_0 \) is that certain students are good against the \( H_1 \) that all the students are good, then the researcher should decide the number of items to be tested and the criteria on the basis of which to accept or reject the hypothesis.

4) **Type I And Type II Errors:**

As regards the testing of hypotheses, a researcher can make basically two types of errors. He/she may reject \( H_0 \) when it is true, or accept \( H_0 \) when it is not true. The former is called as Type I error and the latter is known as Type II error. In other words, Type I error implies the rejection of a hypothesis when it must have been accepted, while Type II error implies the acceptance of a hypothesis which must have been rejected. Type I error is denoted by \( \alpha \) (alpha) and is known as \( \alpha \) error, while Type II error is usually denoted by \( \beta \) (beta) and is known as \( \beta \) error.

5) **One-Tailed And Two-Tailed Tests:**

These two types of tests are very important in the context of hypothesis testing. A two-tailed test rejects the null hypothesis, when the sample mean is significantly greater or lower than the hypothesized value of the mean of the population. Such a test is suitable when the null hypothesis is some specified value, the alternative hypothesis is a value that is not equal to the specified value of the null hypothesis.

1.6.3 **Procedure Of Hypothesis Testing:**

Testing a hypothesis refers to verifying whether the hypothesis is valid or not. Hypothesis testing attempts to check whether to accept or not to accept the null hypothesis. The procedure of hypothesis testing includes all the steps that a researcher undertakes for making a choice between the two alternative actions of rejecting or accepting a null hypothesis. The various steps involved in hypothesis testing are as follows:
1) Making a Formal Statement:

This step involves making a formal statement of the null hypothesis ($H_0$) and the alternative hypothesis ($H_a$). This implies that the hypotheses should be clearly stated within the purview of the research problem. For example, suppose a school teacher wants to test the understanding capacity of the students which must be rated more than 90 per cent in terms of marks, the hypotheses may be stated as follows:

Null Hypothesis $H_0$: $\mu = 100$  Alternative Hypothesis $H_a$: $\mu > 100$

2) Selecting A Significance Level:

The hypotheses should be tested on a pre-determined level of significance, which should be specified. Usually, either 5% level or 1% level is considered for the purpose. The factors that determine the levels of significance are: (a) the magnitude of difference between the sample means; (b) the sample size; (c) the variability of measurements within samples; and (d) whether the hypothesis is directional or non-directional (Kothari, 1988). In sum, the level of significance should be sufficient in the context of the nature and purpose of enquiry.

3) Deciding The Distribution To Use:

After making decision on the level of significance for hypothesis testing, the researcher has to next determine the appropriate sampling distribution. The choice to be made generally relates to normal distribution and the t-distribution. The rules governing the selection of the correct distribution are similar to the ones already discussed with respect to estimation.

4) Selection Of A Random Sample And Computing An Appropriate Value:

Another step involved in hypothesis testing is the selection of a random sample and then computing a suitable value from the sample data relating to test statistic by using the appropriate distribution. In other words, it involves drawing a sample for furnishing empirical data.
5) Calculation Of The Probability:

The next step for the researcher is to calculate the probability that the sample result would diverge as far as it can from expectations, under the situation when the null hypothesis is actually true.

6) Comparing The Probability:

Another step involved consists of making a comparison of the probability calculated with the specified value of α, i.e. The significance level. If the calculated probability works out to be equal to or smaller than the α value in case of one-tailed test, then the null hypothesis is to be rejected. On the other hand, if the calculated probability is greater, then the null hypothesis is to be accepted. In case the null hypothesis \( H_0 \) is rejected, the researcher runs the risk of committing the Type I error. But, if the null hypothesis \( H_0 \) is accepted, then it involves some risk (which cannot be specified in size as long as \( H_0 \) is vague and not specific) of committing the Type II error.

1.7 Sample Survey:

A sample design is a definite plan for obtaining a sample from a given population (Kothari, 1988). Sample constitutes a certain portion of the population or universe. Sampling design refers to the technique or the procedure the researcher adopts for selecting items for the sample from the population or universe. A sample design helps to decide the number of items to be included in the sample, i.e., the size of the sample. The sample design should be determined prior to data collection. There are different kinds of sample designs which a researcher can choose. Some of them are relatively more precise and easier to adopt than the others. A researcher should prepare or select a sample design, which must be reliable and suitable for the research study proposed to be undertaken.

1.8.1 Steps In Sampling Design:

A researcher should take into consideration the following aspects while developing a sample design:
1) **Type Of Universe:**

   The first step involved in developing sample design is to clearly define the number of cases, technically known as the universe. A universe may be finite or infinite. In a finite universe the number of items is certain, whereas in the case of an infinite universe the number of items is infinite (i.e., there is no idea about the total number of items). For example, while the population of a city or the number of workers in a factory comprise finite universes, the number of stars in the sky, or throwing of a dice represent infinite universe.

2) **Sampling Unit:**

   Prior to selecting a sample, decision has to be made about the sampling unit. A sampling unit may be a geographical area like a state, district, village, etc., or a social unit like a family, religious community, school, etc., or it may also be an individual. At times, the researcher would have to choose one or more of such units for his/her study.

3) **Source List:**

   Source list is also known as the ‘sampling frame’, from which the sample is to be selected. The source list consists of names of all the items of a universe. The researcher has to prepare a source list when it is not available. The source list must be reliable, comprehensive, correct, and appropriate. It is important that the source list should be as representative of the population as possible.

4) **Size Of Sample:**

   Size of the sample refers to the number of items to be chosen from the universe to form a sample. For a researcher, this constitutes a major problem. The size of sample must be optimum. An optimum sample may be defined as the one that satisfies the requirements of representativeness, flexibility, efficiency, and reliability. While deciding the size of sample, a researcher should determine the desired precision and the acceptable confidence level for the estimate. The size of the population variance should be considered, because in the case of a larger variance generally a larger sample is required. The size of the population should be considered,
as it also limits the sample size. The parameters of interest in a research study should also be considered, while deciding the sample size. Besides, costs or budgetary constraint also plays a crucial role in deciding the sample size.

(A) Parameters Of Interest:

The specific population parameters of interest should also be considered while determining the sample design. For example, the researcher may want to make an estimate of the proportion of persons with certain characteristic in the population, or may be interested in knowing some average regarding the population. The population may also consist of important sub-groups about whom the researcher would like to make estimates. All such factors have strong impact on the sample design the researcher selects.

(B) Budgetary Constraint:

From the practical point of view, cost considerations exercise a major influence on the decisions related to not only the sample size, but also on the type of sample selected. Thus, budgetary constraint could also lead to the adoption of a non-probability sample design.

(c) Sampling Procedure:

Finally, the researcher should decide the type of sample or the technique to be adopted for selecting the items for a sample. This technique or procedure itself may represent the sample design. There are different sample designs from which a researcher should select one for his/her study. It is clear that the researcher should select that design which, for a given sample size and budget constraint, involves a smaller error.

1.7.2 Criteria For Selecting A Sampling Procedure:

Basically, two costs are involved in a sampling analysis, which govern the selection of a sampling procedure. They are:

1) The cost of data collection, and
2) The cost of drawing incorrect inference from the selected data.
There are two causes of incorrect inferences, namely systematic bias and sampling error. Systematic bias arises out of errors in the sampling procedure. They cannot be reduced or eliminated by increasing the sample size. Utmost, the causes of these errors can be identified and corrected. Generally, a systematic bias arises out of one or more of the following factors:

a. Inappropriate sampling frame,
b. Defective measuring device,
c. Non-respondents,
d. Indeterminacy principle, and
e. Natural bias in the reporting of data.

Sampling error refers to the random variations in the sample estimates around the true population parameters. Because they occur randomly and likely to be equally in either direction, they are of compensatory type, the expected value of which errors tend to be equal to zero. Sampling error tends to decrease with the increase in the size of the sample. It also becomes smaller in magnitude when the population is homogenous.

Sampling error can be computed for a given sample size and design. The measurement of sampling error is known as ‘precision of the sampling plan.’ When the sample size is increased, the precision can be improved. However, increasing the sample size has its own limitations. The large sized sample not only increases the cost of data collection, but also increases the systematic bias. Thus, an effective way of increasing the precision is generally to choose a better sampling design, which has smaller sampling error for a given sample size at a specified cost. In practice, however, researchers generally prefer a less precise design owing to the ease in adopting the same, in addition to the fact that systematic bias can be controlled better way in such designs.

In sum, while selecting the sample, a researcher should ensure that the procedure adopted involves a relatively smaller sampling error and helps to control systematic bias.
1.7.3 Characteristics Of A Good Sample Design:

The following are the characteristic features of a good sample design:

a. The sample design should yield a truly representative sample;
b. The sample design should be such that it results in small sampling error;
c. The sample design should be viable in the context of budgetary constraints of the research study;
d. The sample design should be such that the systematic bias can be controlled; and
e. The sample must be such that the results of the sample study would be applicable, in general, to the universe at a reasonable level of confidence.

1.7.4 Different Types Of Sample Designs:

Sample designs may be classified into different categories based on two factors, namely, the representation basis and the element selection technique. Under the representation basis, the sample may be classified as:

I. Non-probability sampling
II. Probability sampling

While probability sampling is based on random selection, the non-probability sampling is based on ‘non-random’ selection of samples.

I. Non-Probability Sampling:

Non-probability sampling is the sampling procedure that does not afford any basis for estimating the probability that each item in the population would have an equal chance of being included in the sample. Non-probability sampling is also known as deliberate sampling, judgment sampling and purposive sampling. Under this type of sampling, the items for the sample are deliberately chosen by the researcher; and his/her choice concerning the choice of items remains supreme. In other words, under non-probability sampling the researchers select a particular unit of the universe for forming a sample on the basis that the small number that
is thus selected out of a huge one would be typical or representative of the whole population. For example, to study the economic conditions of people living in a state, a few towns or village may be purposively selected for an intensive study based on the principle that they are representative of the entire state. In such a case, the judgment of the researcher of the study assumes prime importance in this sampling design.

**Quota Sampling:**

Quota sampling is also an example of non-probability sampling. Under this sampling, the researchers simply assume quotas to be filled from different strata, with certain restrictions imposed on how they should be selected. This type of sampling is very convenient and is relatively less expensive. However, the samples selected using this method certainly do not satisfy the characteristics of random samples. They are essentially judgment samples and inferences drawn based on that, would not be amenable to statistical treatment in a formal way.

**II. Probability Sampling:**

Probability sampling is also known as ‘choice sampling’ or ‘random sampling’. Under this sampling design, every item of the universe has an equal chance of being included in the sample. In a way, it is a lottery method under which individual units are selected from the whole group, not deliberately, but by using some mechanical process. Therefore, only chance would determine whether an item or the other would be included in the sample or not. The results obtained from probability or random sampling would be assured in terms of probability. That is, the researcher can measure the errors of estimation or the significance of results obtained from the random sample. This is the superiority of random sampling design over the deliberate sampling design. Random sampling satisfies the law of statistical regularity, according to which if on an average the sample chosen is random, then it would have the same composition and characteristics of the universe. This is the reason why the random sampling method is considered the best technique of choosing a representative sample.
The following are the implications of the random sampling:

i. it provides each element in the population an equal probable chance of being chosen in the sample, with all choices being independent of one another and

ii. it offers each possible sample combination an equal probable opportunity of being selected.

1.7.5 Method Of Selecting A Random Sample:

The process of selecting a random sample involves writing the name of each element of a finite population on a slip of paper and putting them into a box or a bag. Then they have to be thoroughly mixed and then the required number of slips for the sample can be picked one after the other without replacement. While doing this, it has to be ensured that in successive drawings each of the remaining elements of the population has an equal chance of being chosen. This method results in the same probability for each possible sample.

1.7.6 Complex Random Sampling Designs:

Under restricted sampling technique, the probability sampling may result in complex random sampling designs. Such designs are known as mixed sampling designs. Many of such designs may represent a combination of non-probability and probability sampling procedures in choosing a sample.

Some of the prominent complex random sampling designs are as follows:

(i) Systematic Sampling:

In some cases, the best way of sampling is to select every first item on a list. Sampling of this kind is called as systematic sampling. An element of randomness is introduced in this type of sampling by using random numbers to select the unit with which to start. For example, if a 10 per cent sample is required out of 100 items, the first item would be selected randomly from the first low of item and thereafter every 10th item. In this kind of sampling, only the first unit is selected randomly, while rest of the units of the sample is chosen at fixed intervals.
(ii) **Stratified Sampling:**

When a population from which a sample is to be selected does not comprise a homogeneous group, stratified sampling technique is generally employed for obtaining a representative sample. Under stratified sampling, the population is divided into many sub-populations in such a manner that they are individually more homogeneous than the rest of the total population. Then, items are selected from each stratum to form a sample. As each stratum is more homogeneous than the remaining total population, the researcher is able to obtain a more precise estimate for each stratum and by estimating each of the component parts more accurately; he/she is able to obtain a better estimate of the whole. In sum, stratified sampling method yields more reliable and detailed information.

(iii) **Cluster Sampling:**

When the total area of research interest is large, a convenient way in which a sample can be selected is to divide the area into a number of smaller non-overlapping areas and then randomly selecting a number of such smaller areas. In the process, the ultimate sample would consist of all the units in these small areas or clusters. Thus in cluster sampling, the total population is sub-divided into numerous relatively smaller subdivisions, which in themselves constitute clusters of still smaller units. And then, some of such clusters are randomly chosen for inclusion in the overall sample.

(iv) **Area Sampling:**

When clusters are in the form of some geographic subdivisions, then cluster sampling is termed as area sampling. That is, when the primary sampling unit represents a cluster of units based on geographic area, the cluster designs are distinguished as area sampling. The merits and demerits of cluster sampling are equally applicable to area sampling.

(v) **Multi-Stage Sampling:**

A further development of the principle of cluster sampling is multi-stage sampling. When the researcher desires to investigate the working efficiency of nationalized banks in India and a sample of few
banks is required for this purpose, the first stage would be to select large primary sampling unit like the states in the country. Next, certain districts may be selected and all banks interviewed in the chosen districts. This represents a two-stage sampling design, with the ultimate sampling units being clusters of districts.

On the other hand, if instead of taking census of all banks within the selected districts, the researcher chooses certain towns and interviews all banks in it, this would represent three-stage sampling design. Again, if instead of taking a census of all banks within the selected towns, the researcher randomly selects sample banks from each selected town, then it represents a case of using a four-stage sampling plan. Thus, if the researcher selects randomly at all stages, then it is called as multi-stage random sampling design.

(vi) **Sampling With Probability Proportional To Size:**

When the case of cluster sampling units does not have exactly or approximately the same number of elements, it is better for the researcher to adopt a random selection process, where the probability of inclusion of each cluster in the sample tends to be proportional to the size of the cluster. For this, the number of elements in each cluster has to be listed, irrespective of the method used for ordering it. Then the researcher should systematically pick the required number of elements from the cumulative totals. The actual numbers thus chosen would not however reflect the individual elements, but would indicate as to which cluster and how many from them are to be chosen by using simple random sampling or systematic sampling. The outcome of such sampling is equivalent to that of simple random sample. The method is also less cumbersome and is also relatively less expensive.

Thus, a researcher has to pass through various stages of conducting research once the problem of interest has been selected. Research methodology familiarizes a researcher with the complex scientific methods of conducting research, which yield reliable results that are useful to policy-makers, government, industries etc. in decision-making.
References:

5. Pauline V. Young, *Scientific Social Surveys and Research*, p.30

Questions:

1. Define research.
2. What are the objectives of research?
3. State the significance of research.
4. What is the importance of knowing how to do research?
5. Briefly outline research process.
6. Highlight the different research approaches.
7. Discuss the qualities of a researcher.
8. Explain the different types of research.
9. What is a research problem?
10. Outline the features of research design.
11. Discuss the features of a good research design.
12. Describe the different types of research design.
13. Explain the significance of research design.
14. What is a case study?
15. Discuss the criteria for evaluating case study.
16. Define hypothesis.
17. What are the characteristic features of a hypothesis?
18. Distinguish between null and alternative hypothesis.
19. Differentiate type i error and type ii error.
20. How is a hypothesis tested?
21. Define the concept of sampling design.
22. Describe the steps involved in sampling design.
23. Discuss the criteria for selecting a sampling procedure.
24. Distinguish between probability and non-probability sampling.
25. How is a random sample selected?
26. Explain complex random sampling designs.
UNIT—II

Lesson 1: Data Collection & Sources Of Data

Lesson Outline:

› Primary Data, Secondary Data
› Investigation
› Indirect Oral Methods Of Collecting Primary Data
› Direct Personal Interviews
› Information Received Through Local Agencies
› Mailed Questionnaire Method
› Schedules Sent Through Enumerators

Learning Objectives:

› After reading this lesson, you should be able to understand the
› Meaning of primary data, Secondary data
› Preliminaries of data collection
› Method of data collection
› Methods of collecting primary data
› Usefulness of primary data
› Merits and demerits of different methods of primary data collection
› Precautions while collecting primary data.
› Secondary data collection

Introduction:

It is important for a researcher to know the sources of data which he requires for different purposes. Data are nothing but the information. There are two sources of information or data they are - Primary and Secondary data. The data are name after the source. Primary data refers to the data collected for the first time, whereas secondary data refers to the data that have already been collected and used earlier by somebody or some agency. For example, the statistics collected by the Government
of India relating to the population is primary data for the Government of India since it has been collected for the first time. Later when the same data are used by a researcher for his study of a particular problem, then the same data become the secondary data for the researcher. Both the sources of information have their merits and demerits. The selection of a particular source depends upon the (a) purpose and scope of enquiry, (b) availability of time, (c) availability of finance, (d) accuracy required, (e) statistical tools to be used, (f) sources of information (data), and (g) method of data collection.

(a) **Purpose And Scope Of Enquiry:**

The purpose and scope of data collection or survey should be clearly set out at the very beginning. It requires the clear statement of the problem indicating the type of information which is needed and the use for which it is needed. If for example, the researcher is interested in knowing the nature of price change over a period of time, it would be necessary to collect data of commodity prices. It must be decided whether it would be helpful to study wholesale or retail prices and the possible uses to which such information could be put. The objective of an enquiry may be either to collect specific information relating to a problem or adequate data to test a hypothesis. Failure to set out clearly the purpose of enquiry is bound to lead to confusion and waste of resources.

After the purpose of enquiry has been clearly defined, the next step is to decide about the scope of the enquiry. Scope of the enquiry means the coverage with regard to the type of information, the subject-matter and geographical area. For instance, an enquiry may relate to India as a whole or a state or an industrial town wherein a particular problem related to a particular industry can be studied.

(b) **Availability Of Time:**

The investigation should be carried out within a reasonable period of time, failing which the information collected may become outdated, and would have no meaning at all. For instance, if a producer wants to know the expected demand for a product newly launched by him and the result of the enquiry that the demand would be meager takes two years to reach him, then the whole purpose of enquiry would become useless
because by that time he would have already incurred a huge loss. Thus, in this respect the information is quickly required and hence the researcher has to choose the type of enquiry accordingly.

(c) **Availability Of Resources:**

The investigation will greatly depend on the resources available like number of skilled personnel, the financial position etc. If the number of skilled personnel who will carry out the enquiry is quite sufficient and the availability of funds is not a problem, then enquiry can be conducted over a big area covering a good number of samples, otherwise a small sample size will do.

(d) **The Degree Of Accuracy Desired:**

Deciding the degree of accuracy required is a must for the investigator, because absolute accuracy in statistical work is seldom achieved. This is so because (i) statistics are based on estimates, (ii) tools of measurement are not always perfect and (iii) there may be unintentional bias on the part of the investigator, enumerator or informant. Therefore, a desire of 100% accuracy is bound to remain unfulfilled. Degree of accuracy desired primarily depends upon the object of enquiry. For example, when we buy gold, even a difference of 1/10th gram in its weight is significant, whereas the same will not be the case when we buy rice or wheat. However, the researcher must aim at attaining a higher degree of accuracy, otherwise the whole purpose of research would become meaningless.

(e) **Statistical Tools To Be Used:**

A well defined and identifiable object or a group of objects with which the measurements or counts in any statistical investigation are associated is called a *statistical unit*. For example, in socio-economic survey the unit may be an individual, a family, a household or a block of locality. A very important step before the collection of data begins is to define clearly the statistical units on which the data are to be collected. In number of situations the units are conventionally fixed like the physical units of measurement, such as meters, kilometers, quintals, hours, days, weeks etc., which are well defined and do not need any elaboration or explanation. However, in many statistical investigations, particularly relating to socio-
economic studies, arbitrary units are used which must be clearly defined. This is a must because in the absence of a clear cut and precise definition of the statistical units, serious errors in the data collection may be committed in the sense that we may collect irrelevant data on the items, which should have, in fact, been excluded and omit data on certain items which should have been included. This will ultimately lead to fallacious conclusions.

(f) **Sources Of Information (Data):**

After deciding about the unit, a researcher has to decide about the source from which the information can be obtained or collected. For any statistical inquiry, the investigator may collect the data first hand or he may use the data from other published sources, such as publications of the government/semi-government organizations or journals and magazines etc.

(g) **Method Of Data Collection:**

There is no problem if secondary data are used for research. However, if primary data are to be collected, a decision has to be taken whether (i) census method or (ii) sampling technique is to be used for data collection. In census method, we go for total enumeration i.e., all the units of a universe have to be investigated. But in sampling technique, we inspect or study only a selected representative and adequate fraction of the population and after analyzing the results of the sample data we draw conclusions about the characteristics of the population. Selection of a particular technique becomes difficult because where population or census method is more scientific and 100% accuracy can be attained through this method, choosing this becomes difficult because it is time taking, it requires more labor and it is very expensive. Therefore, for a single researcher or for a small institution it proves to be unsuitable. On the other hand, sample method is less time taking, less laborious and less expensive but a 100% accuracy cannot be attained through this method because of sampling and non-sampling errors attached to this method. Hence, a researcher has to be very cautious and careful while choosing a particular method.
Methods of Collecting Primary Data:

Primary data may be obtained by applying any of the following methods:

1. Direct Personal Interviews.
2. Indirect Oral Interviews.
3. Information from Correspondents.
5. Schedule Sent Through Enumerators.

1. Direct Personal Interviews:

A face to face contact is made with the informants (persons from whom the information is to be obtained) under this method of collecting data. The interviewer asks them questions pertaining to the survey and collects the desired information. Thus, if a person wants to collect data about the working conditions of the workers of the Tata Iron and Steel Company, Jamshedpur, he would go to the factory, contact the workers and obtain the desired information. The information collected in this manner is first hand and also original in character. There are many merits and demerits of this method, which are discussed as under:

Merits:

1. Most often respondents are happy to pass on the information required from them when contacted personally and thus response is encouraging.
2. The information collected through this method is normally more accurate because interviewer can clear doubts of the informants about certain questions and thus obtain correct information. In case the interviewer apprehends that the informant is not giving accurate information, he may cross-examine him and thereby try to obtain the information.
3. This method also provides the scope for getting supplementary information from the informant, because while interviewing it is possible to ask some supplementary questions which may be of greater use later.
4. There might be some questions which the interviewer would find difficult to ask directly, but with some tactfulness, he can mingle such
questions with others and get the desired information. He can twist the questions keeping in mind the informant’s reaction. Precisely, a delicate situation can usually be handled more effectively by a personal interview than by other survey techniques.

5. The interviewer can adjust the language according to the status and educational level of the person interviewed, and thereby can avoid inconvenience and misinterpretation on the part of the informant.

**Demerits:**

1. This method can prove to be expensive if the number of informants is large and the area is widely spread.
2. There is a greater chance of personal bias and prejudice under this method as compared to other methods.
3. The interviewers have to be thoroughly trained and experienced; otherwise they may not be able to obtain the desired information. Untrained or poorly trained interviewers may spoil the entire work.
4. This method is more time taking as compared to others. This is because interviews can be held only at the convenience of the informants. Thus, if information is to be obtained from the working members of households, interviews will have to be held in the evening or on week end. Even during evening only an hour or two can be used for interviews and hence, the work may have to be continued for a long time, or a large number of people may have to be employed which may involve huge expenses.

**Conclusion:**

Though there are some demerits in this method of data collection still we cannot say that it is not useful. The matter of fact is that this method is suitable for intensive rather than extensive field surveys. Hence, it should be used only in those cases where intensive study of a limited field is desired.

In the present time of extreme advancement in the communication system, the investigator instead of going personally and conducting a face to face interview may also obtain information over telephone. A good number of surveys are being conducted every day by newspapers and television channels by sending the reply either by e-mail or SMS. This
method has become very popular nowadays as it is less expensive and the response is extremely quick. But this method suffers from some serious defects, such as (a) those who own a phone or a television only can be approached by this method, (b) only few questions can be asked over phone or through television, (c) the respondents may give a vague and reckless answers because answers on phone or through SMS would have to be very short.

2. **Indirect Oral Interviews:**

Under this method of data collection, the investigator contacts third parties generally called 'witnesses' who are capable of supplying necessary information. This method is generally adopted when the information to be obtained is of a complex nature and informants are not inclined to respond if approached directly. For example, when the researcher is trying to obtain data on drug addiction or the habit of taking liquor, there is high probability that the addicted person will not provide the desired data and hence will disturb the whole research process. In this situation taking the help of such persons or agencies or the neighbours who know them well becomes necessary. Since these people know the person well, they can provide the desired data. Enquiry Committees and Commissions appointed by the Government generally adopt this method to get people's views and all possible details of the facts related to the enquiry.

Though this method is very popular, its correctness depends upon a number of factors such as

1. The person or persons or agency whose help is solicited must be of proven integrity; otherwise any bias or prejudice on their part will not bring out the correct information and the whole process of research will become useless.
2. The ability of the interviewers to draw information from witnesses by means of appropriate questions and cross-examination.
3. It might happen that because of bribery, nepotism or certain other reasons those who are collecting the information give it such a twist that correct conclusions are not arrived at.

Therefore, for the success of this method it is necessary that the evidence of one person alone is not relied upon. Views from other persons
and related agencies should also be ascertained to find the real position. Utmost care must be exercised in the selection of these persons because it is on their views that the final conclusions are reached.

3. Information from Correspondents:

The investigator appoints local agents or correspondents in different places to collect information under this method. These correspondents collect and transmit the information to the central office where data are processed. This method is generally adopted by newspaper agencies. Correspondents who are posted at different places supply information relating to such events as accidents, riots, strikes, etc., to the head office. The correspondents are generally paid staff or sometimes they may be honorary correspondents also. This method is also adopted generally by the government departments in such cases where regular information is to be collected from a wide area. For example, in the construction of a wholesale price index numbers regular information is obtained from correspondents appointed in different areas. The biggest advantage of this method is that, it is cheap and appropriate for extensive investigation. But a word of caution is that it may not always ensure accurate results because of the personal prejudice and bias of the correspondents. As stated earlier, this method is suitable and adopted in those cases where the information is to be obtained at regular intervals from a wide area.

4. Mailed Questionnaire Method:

Under this method, a list of questions pertaining to the survey which is known as ‘Questionnaire’ is prepared and sent to the various informants by post. Sometimes the researcher himself too contacts the respondents and gets the responses related to various questions in the questionnaire. The questionnaire contains questions and provides space for answers. A request is made to the informants through a covering letter to fill up the questionnaire and send it back within a specified time. The questionnaire studies can be classified on the basis of:

i. The degree to which the questionnaire is formalized or structured.
ii. The disguise or lack of disguise of the questionnaire and
iii. The communication method used.
When no formal questionnaire is used, interviewers adapt their questioning to each interview as it progresses. They might even try to elicit responses by indirect methods, such as showing pictures on which the respondent comments. When a researcher follows a prescribed sequence of questions, it is referred to as structured study. On the other hand, when no prescribed sequence of questions exists, the study is non-structured.

When questionnaires are constructed in such a way that the objective is clear to the respondents then these questionnaires are known as non-disguised; on the other hand, when the objective is not clear, the questionnaire is a disguised one. On the basis of these two classifications, four types of studies can be distinguished:

1. Non-disguised structured,
2. Non-disguised non-structured,
3. Disguised structured and
4. Disguised non-structured.

There are certain merits and demerits of this method of data collection which are discussed below:

Merits:

1. Questionnaire method of data collection can be easily adopted where the field of investigation is very vast and the informants are spread over a wide geographical area.
2. This method is relatively cheap and expeditious provided the informants respond in time.
3. This method has proved to be superior when compared to other methods like personal interviews or telephone method. This is because when questions pertaining to personal nature or the ones requiring reaction by the family are put forth to the informants, there is a chance for them to be embarrassed in answering them.

Demerits:

1. This method can be adopted only where the informants are literates so that they can understand written questions and lend the answers in writing.
2. It involves some uncertainty about the response. Co-operation on the part of informants may be difficult to presume.

3. The information provided by the informants may not be correct and it may be difficult to verify the accuracy.

However, by following the guidelines given below, this method can be made more effective:

The questionnaires should be made in such a manner that they do not become an undue burden on the respondents; otherwise the respondents may not return them back.

i. Prepaid postage stamp should be affixed

ii. The sample should be large

iii. It should be adopted in such enquiries where it is expected that the respondents would return the questionnaire because of their own interest in the enquiry.

iv. It should be preferred in such enquiries where there could be a legal compulsion to provide the information.

5. **Schedules Sent Through Enumerators:**

Another method of data collection is sending schedules through the enumerators or interviewers. The enumerators contact the informants, get replies to the questions contained in a schedule and fill them in their own handwriting in the questionnaire form. There is difference between questionnaire and schedule. Questionnaire refers to a device for securing answers to questions by using a form which the respondent fills in himself, whereas schedule is the name usually applied to a set of questions which are asked in a face-to-face situation with another person. This method is free from most of the limitations of the mailed questionnaire method.

**Merits:**

The main merits or advantages of this method are listed below:

1. It can be adopted in those cases where informants are illiterate.

2. There is very little scope of non-response as the enumerators go personally to obtain the information.
3. The information received is more reliable as the accuracy of statements can be checked by supplementary questions wherever necessary.

This method too like others is not free from defects or limitations. The main limitations are listed below:

**Demerits:**

1. In comparison to other methods of collecting primary data, this method is quite costly as enumerators are generally paid persons.
2. The success of the method depends largely upon the training imparted to the enumerators.
3. Interviewing is a very skilled work and it requires experience and training. Many statisticians have the tendency to neglect this extremely important part of the data collecting process and this result in bad interviews. Without good interviewing most of the information collected may be of doubtful value.
4. Interviewing is not only a skilled work but it also requires a great degree of politeness and thus the way the enumerators conduct the interview would affect the data collected. When questions are asked by a number of different interviewers, it is possible that variations in the personalities of the interviewers will cause variation in the answers obtained. This variation will not be obvious. Hence, every effort must be made to remove as much of variation as possible due to different interviewers.

**Secondary Data:**

As stated earlier, secondary data are those data which have already been collected and analyzed by some earlier agency for its own use, and later the same data are used by a different agency. According to W.A. Neiswanger, “A primary source is a publication in which the data are published by the same authority which gathered and analyzed them. A secondary source is a publication, reporting the data which was gathered by other authorities and for which others are responsible.”
Sources Of Secondary Data:

The various sources of secondary data can be divided into two broad categories:
1. Published sources, and
2. Unpublished sources.

1. Published Sources:

The governmental, international and local agencies publish statistical data, and chief among them are explained below:

(a) International Publications:

There are some international institutions and bodies like I.M.F, I.B.R.D, I.C.A.F.E and U.N.O who publish regular and occasional reports on economic and statistical matters.

(b) Official Publications of Central and State Governments:

Several departments of the Central and State Governments regularly publish reports on a number of subjects. They gather additional information. Some of the important publications are: The Reserve Bank of India Bulletin, Census of India, Statistical Abstracts of States, Agricultural Statistics of India, Indian Trade Journal, etc.

(c) Semi-Official Publications:

Semi-Government institutions like Municipal Corporations, District Boards, Panchayats, etc. Publish reports relating to different matters of public concern.

(d) Publications of Research Institutions:

Indian Statistical Institute (I.S.I), Indian Council of Agricultural Research (I.C.A.R), Indian Agricultural Statistics Research Institute (I.A.S.R.I), etc. Publish the findings of their research programmes.

(e) Publications of various Commercial and Financial Institutions
(f) Reports of various Committees and Commissions appointed by the Government as the Raj Committee’s Report on Agricultural Taxation, Wanchoo Committee’s Report on Taxation and Black Money, etc. Are also important sources of secondary data.

(g) Journals and News Papers:

Journals and News Papers are very important and powerful source of secondary data. Current and important materials on statistics and socio-economic problems can be obtained from journals and newspapers like Economic Times, Commerce, Capital, Indian Finance, Monthly Statistics of trade etc.

2. Unpublished Sources:

Unpublished data can be obtained from many unpublished sources like records maintained by various government and private offices, the theses of the numerous research scholars in the universities or institutions etc.

Precautions In The Use Of Secondary Data:

Since secondary data have already been obtained, it is highly desirable that a proper scrutiny of such data is made before they are used by the investigator. In fact the user has to be extra-cautious while using secondary data. In this context Prof. Bowley rightly points out that “Secondary data should not be accepted at their face value.” The reason being that data may be erroneous in many respects due to bias, inadequate size of the sample, substitution, errors of definition, arithmetical errors etc. Even if there is no error such data may not be suitable and adequate for the purpose of the enquiry. Prof. SimonKuznet’s view in this regard is also of great importance. According to him, “the degree of reliability of secondary source is to be assessed from the source, the compiler and his capacity to produce correct statistics and the users also, for the most part, tend to accept a series particularly one issued by a government agency at its face value without enquiring its reliability”.

Therefore, before using the secondary data the investigators should consider the following factors:
4. The Suitability Of Data:

The investigator must satisfy himself that the data available are suitable for the purpose of enquiry. It can be judged by the nature and scope of the present enquiry with the original enquiry. For example, if the object of the present enquiry is to study the trend in retail prices, and if the data provide only wholesale prices, such data are unsuitable.

(A) Adequacy Of Data:

If the data are suitable for the purpose of investigation then we must consider whether the data are useful or adequate for the present analysis. It can be studied by the geographical area covered by the original enquiry. The time for which data are available is very important element. In the above example, if our object is to study the retail price trend of India, and if the available data cover only the retail price trend in the state of Bihar, then it would not serve the purpose.

(b) Reliability Of Data:

The reliability of data is must. Without which there is no meaning in research. The reliability of data can be tested by finding out the agency that collected such data. If the agency has used proper methods in collection of data, statistics may be relied upon.

It is not enough to have baskets of data in hand. In fact, data in a raw form are nothing but a handful of raw material waiting for proper processing so that they can become useful. Once data have been obtained from primary or secondary source, the next step in a statistical investigation is to edit the data i.e. To scrutinize the same. The chief objective of editing is to detect possible errors and irregularities. The task of editing is a highly specialized one and requires great care and attention. Negligence in this respect may render useless the findings of an otherwise valuable study. Editing data collected from internal records and published sources is relatively simple but the data collected from a survey need excessive editing.

While editing primary data, the following considerations should be borne in mind:

1. The data should be complete in every respect
2. The data should be accurate
3. The data should be consistent, and
4. The data should be homogeneous.

Data to possess the above mentioned characteristics have to undergo the same type of editing which is discussed below:

5. **Editing for Completeness:**

   while editing, the editor should see that each schedule and questionnaire is complete in all respects. He should see to it that the answers to each and every question have been furnished. If some questions are not answered and if they are of vital importance, the informants should be contacted again either personally or through correspondence. Even after all the efforts it may happen that a few questions remain unanswered. In such questions, the editor should mark ‘No answer’ in the space provided for answers and if the questions are of vital importance then the schedule or questionnaire should be dropped.

(a) **Editing for Consistency:**

   At the time of editing the data for consistency, the editor should see that the answers to questions are not contradictory in nature. If they are mutually contradictory answers, he should try to obtain the correct answers either by referring back the questionnaire or by contacting, wherever possible, the informant in person. For example, if amongst others, two questions in questionnaire are (a) Are you a student? (b) Which class do you study and the reply to the first question is ‘no’ and to the latter ‘tenth’ then there is contradiction and it should be clarified.

(b) **Editing for Accuracy:**

   The reliability of conclusions depends basically on the correctness of information. If the information supplied is wrong, conclusions can never be valid. It is, therefore, necessary for the editor to see that the information is accurate in all respects. If the inaccuracy is due to arithmetical errors, it can be easily detected and corrected. But if the cause of inaccuracy is faulty information supplied, it may be difficult to verify it and an example of this kind is information relating to income, age etc.
(c) Editing For Homogeneity:

Homogeneity means the condition in which all the questions have been understood in the same sense. The editor must check all the questions for uniform interpretation. For example, as to the question of income, if some informants have given monthly income, others annual income and still others weekly income or even daily income, no comparison can be made. Therefore, it becomes an essential duty of the editor to check up that the information supplied by the various people is homogeneous and uniform.

Choice Between Primary and Secondary Data:

As we have already seen, there are a lot of differences in the methods of collecting Primary and Secondary data. Primary data which is to be collected originally involves an entire scheme of plan starting with the definitions of various terms used, units to be employed, type of enquiry to be conducted, extent of accuracy aimed at etc. For the collection of secondary data, a mere compilation of the existing data would be sufficient. A proper choice between the type of data needed for any particular statistical investigation is to be made after taking into consideration the nature, objective and scope of the enquiry; the time and the finances at the disposal of the agency; the degree of precision aimed at and the status of the agency (whether government- state or central-or private institution of an individual).

In using the secondary data, it is best to obtain the data from the primary source as far as possible. By doing so, we would at least save ourselves from the errors of transcription which might have inadvertently crept in the secondary source. Moreover, the primary source will also provide us with detailed discussion about the terminology used, statistical units employed, size of the sample and the technique of sampling (if sampling method was used), methods of data collection and analysis of results and we can ascertain ourselves if these would suit our purpose.

Now-a-days in a large number of statistical enquiries, secondary data are generally used because fairly reliable published data on a large number of diverse fields are now available in the publications of governments, private organizations and research institutions, agencies, periodicals and magazines etc. In fact, primary data are collected only if there do not exist
any secondary data suited to the investigation under study. In some of the investigations both primary as well as secondary data may be used.

Summary:

There are two types of data, primary and secondary. Data which are collected first hand are called Primary data and data which have already been collected and used by somebody are called Secondary data. There are two methods of collecting data: (a) Survey method or total enumeration method and (b) Sample method. When a researcher goes for investigating all the units of the subject, it is called as survey method. On the other hand if he/she resorts to investigating only a few units of the subject and gives the result on the basis of that, it is known as sample survey method. There are different sources of collecting Primary and Secondary data. Some of the important sources of Primary data are—Direct Personal Interviews, Indirect Oral Interviews, Information from Correspondents, Mailed questionnaire method, Schedules sent through enumerators and so on. Though all these sources or methods of Primary data have their relative merits and demerits, a researcher should use a particular method with lot of care. There are basically two sources of collecting secondary data- (a) Published sources and (b) Unpublished sources. Published sources are like publications of different government and semi-government departments, research institutions and agencies etc. Whereas unpublished sources are like records maintained by different government departments and unpublished theses of different universities etc. Editing of secondary data is necessary for different purposes as – editing for completeness, editing for consistency, editing for accuracy and editing for homogeneity.

It is always a tough task for the researcher to choose between primary and secondary data. Though primary data are more authentic and accurate, time, money and labor involved in obtaining these more often prompt the researcher to go for the secondary data. There are certain amount of doubt about its authenticity and suitability, but after the arrival of many government and semi government agencies and some private institutions in the field of data collection, most of the apprehensions in the mind of the researcher have been removed.
Self Assessment Questions (SAQS):

a) Explain primary and secondary data and distinguish between them.  
   (Refer the introduction part of this lesson.)

b) Explain the different methods of collecting primary data.  
   (Explain direct personal, indirect oral interview, information received  
   through agencies etc.)

c) Explain the merits and demerits of different methods of  
   collecting primary data.  
   (Refer the methods of collecting primary data)

d) Explain the different sources of secondary data and the  
   precautions in using secondary data.

e) What is editing of secondary data? Why is it required?

f) What are the different types of editing of secondary data?

GLOSSARY OF TERMS:

Primary Source:  
It is one that itself collects the data.

Secondary Source:  
It is one that makes available data collected by some other agency.

Collection of Statistics:  
Collection means the assembling for the purpose of particular  
investigation of entirely new data presumably not already available in  
published sources.

Questionnaire:  
A list of questions properly selected and arranged pertaining to the  
investigation.

Investigator:  
Investigator is a person who collects the information.

Respondent:  
A person who fills the questionnaire or provides the required  
information.
UNIT II

Lesson 2: Questionnaire & Sampling

Lesson Outline

- Meaning Of Questionnaire.
- Drafting Of Questionnaire.
- Size Of Questions
- Clarity Of Questions
- Logical Sequence Of Questions
- Simple Meaning Questions
- Other Requirements Of A Good Questionnaire
- Meaning And Essentials Of Sampling.

Learning Objectives

After reading this lesson you should be able to

- Understand the meaning of questionnaire
- Different requirements and characteristics of a good questionnaire
- Meaning of sampling
- Essentials of sampling

Introduction:

Nowadays questionnaire is widely used for data collection in social research. It is a reasonably fair tool for gathering data from large, diverse, varied and scattered social groups. The questionnaire is the media of communication between the investigator and the respondents. According to Bogardus, a questionnaire is a list of questions sent to a number of persons for their answers and which obtains standardized results that can be tabulated and treated statistically. The Dictionary of Statistical Terms defines it as a “group of or sequence of questions designed to elicit
information upon a subject or sequence of subjects from information.” A questionnaire should be designed or drafted with utmost care and caution so that all the relevant and essential information for the enquiry may be collected without any difficulty, ambiguity and vagueness. Drafting of a good questionnaire is a highly specialized job and requires great care skill, wisdom, efficiency and experience. No hard and fast rule can be laid down for designing or framing a questionnaire. However, in this connection, the following general points may be borne in mind:

1. **Size Of The Questionnaire Should Be Small:**

   A researcher should try his best to keep the number of questions as small as possible, keeping in view the nature, objectives and scope of the enquiry. Respondent’s time should not be wasted by asking irrelevant and unimportant questions. A large number of questions would involve more work for the investigator and thus result in delay on his part in collecting and submitting the information. A large number of unnecessary questions may annoy the respondent and he may refuse to cooperate. A reasonable questionnaire should contain from 15 to 25 questions at large. If a still larger number of questions are a must in any enquiry, then the questionnaire should be divided into various sections or parts.

2. **The Questions Should Be Clear:**

   The questions should be easy, brief, unambiguous, non-offending, courteous in tone, corroborative in nature and to the point, so that much scope of guessing is left on the part of the respondents.

3. **The Questions Should Be Arranged In A Logical Sequence:**

   Logical arrangement of questions reduces lot of unnecessary work on the part of the researcher because it not only facilitates the tabulation work but also does not leave any chance for omissions or commissions. For example, to find if a person owns a television, the logical order of questions would be: Do you own a television? When did you buy it? What is its make? How much did it cost you? Is its performance satisfactory? Have you ever got it serviced?
4. **Questions Should Be Simple To Understand:**

The vague words like good, bad, efficient, sufficient, prosperity, rarely, frequently, reasonable, poor, rich etc., should not be used since these may be interpreted differently by different persons and as such might give unreliable and misleading information. Similarly the use of words having double meaning like price, assets, capital income etc., should also be avoided.

5. **Questions Should Be Comprehensive & Easily Answerable:**

Questions should be designed in such a way that they are readily comprehensible and easy to answer for the respondents. They should not be tedious nor should they tax the respondents’ memory. At the same time questions involving mathematical calculations like percentages, ratios etc., should not be asked.

6. **Questions Of Personal & Sensitive Nature Should Not Be Asked:**

There are some questions which disturb the respondents and he/she may be shy or irritated by hearing such questions. Therefore, every effort should be made to avoid such questions. For example, ‘do you cook yourself or your wife cooks?’ ‘Or do you drink?’ Such questions will certainly irk the respondents and thus be avoided at any cost. If unavoidable then highest amount of politeness should be used.

7. **Types Of Questions:**

Under this head, the questions in the questionnaire may be classified as follows:

(a) **Shut Questions:**

Shut questions are those where possible answers are suggested by the framers of the questionnaire and the respondent is required to tick one of them. Shut questions can further be subdivided into the following forms:
(i) **Simple Alternate Questions:**

In this type of questions the respondent has to choose from the two clear cut alternatives like ‘Yes’ or ‘No’, ‘Right or Wrong’ etc. Such questions are also called as *dichotomous questions*. This technique can be applied with elegance to situations where two clear cut alternatives exist.

(ii) **Multiple Choice Questions:**

Many a times it becomes difficult to define a clear cut alternative and accordingly in such a situation additional answers between Yes and No, like Do not know, No opinion, Occasionally, Casually, Seldom etc., are added. For example, in order to find if a person smokes or drinks, the following multiple choice answers may be used:

Do you smoke?

(a) Yes regularly [ ] (b) No never [ ]
(c) Occasionally [ ] (d) Seldom [ ]

Multiple choice questions are very easy and convenient for the respondents to answer. Such questions save time and also facilitate tabulation. This method should be used if only a selected few alternative answers exist to a particular question.

8. **Leading Questions Should Be Avoided:**

Questions like ‘why do you use a particular type of car, say Maruti car’ should preferably be framed into two questions-

(i) which car do you use? (ii) why do you prefer it?

- It gives smooth ride [ ]
- It gives more mileage [ ]
- It is cheaper [ ]
- It is maintenance free [ ]
9 Cross Checks:

The questionnaire should be so designed as to provide internal checks on the accuracy of the information supplied by the respondents by including some connected questions at least with respect to matters which are fundamental to the enquiry.

10 Pre Testing The Questionnaire:

It would be practical in every sense to try out the questionnaire on a small scale before using it for the given enquiry on a large scale. This has been found extremely useful in practice. The given questionnaire can be improved or modified in the light of the drawbacks, shortcomings and problems faced by the investigator in the pre test.

11 A Covering Letter:

A covering letter from the organizers of the enquiry should be enclosed along with the questionnaire for the purposes regarding definitions, units, concepts used in the questionnaire, for taking the respondent’s confidence, self addressed envelop in case of mailed questionnaire, mention about award or incentives for the quick response, a promise to send a copy of the survey report etc.

SAMPLING

Though sampling is not new, the sampling theory has been developed recently. People knew or not but they have been using the sampling technique in their day to day life. For example a house wife tests a small quantity of rice to see whether it has been well-cooked and gives the generalized result about the whole rice boiling in the vessel. The result arrived at is most of the times 100% correct. In another example, when a doctor wants to examine the blood for any deficiency, takes only a few drops of blood of the patient and examines. The result arrived at is most of the times correct and represent the whole amount of blood available in the body of the patient. In all these cases, by inspecting a few, they simply believe that the samples give a correct idea about the population. Most of our decision are based on the examination of a few items only i.e. Sample studies. In the words of Croxton and Cowdon, “It may be too expensive
or too time consuming to attempt either a complete or a nearly complete coverage in a statistical study. Further to arrive at valid conclusions, it may not be necessary to enumerate all or nearly all of a population. We may study a sample drawn from the large population and if that sample is adequately representative of the population, we should be able to arrive at valid conclusions.”

According to Rosander, “The sample has many advantages over a census or complete enumeration. If carefully designed, the sample is not only considerably cheaper but may give results which are just accurate and sometimes more accurate than those of a census. Hence a carefully designed sample may actually be better than a poorly planned and executed census.”

Merits:

1. **It saves time:**

   Sampling method of data collection saves time because fewer items are collected and processed. When the results are urgently required, this method is very helpful.

2. **It reduces cost:**

   Since only a few and selected items are studied in sampling, there is reduction in cost of money and reduction in terms of man hours.

3. **More reliable results can be obtained:**

   Through sampling, more reliable results can be obtained because (a) there are fewer chances of sampling statistical errors. If there is sampling error, it is possible to estimate and control the results. (b) Highly experienced and trained persons can be employed for scientific processing and analyzing of relatively limited data and they can use their high technical knowledge and get more accurate and reliable results.
4. **It provides more detailed information:**

   As it saves time, money and labor, more detail information can be collected in a sample survey.

5. **Sometimes only sampling method to depend upon:**

   Some times it so happens that one has to depend upon sampling method alone because if the population under study is finite, sampling method is the only method to be used. For example, if someone's blood has to be examined, it will become fatal to take all the blood out from the body and study depending upon the total enumeration method.

6. **Administrative convenience:**

   The organization and administration of sample survey are easy for the reasons which have been discussed earlier.

7. **More scientific:**

   Since the methods used to collect data are based on scientific theory and results obtained can be tested, sampling is a more scientific method of collecting data.

It is not that sampling is free from demerits or shortcomings. There are certain **shortcomings of this method** which are discussed below:

1. **Illusory conclusion:**

   If a sample enquiry is not carefully planned and executed, the conclusions may be inaccurate and misleading.

2. **Sample Not Representative:**

   To make the sample representative is a difficult task. If a representative sample is taken from the universe, the result is applicable to the whole population. If the sample is not representative of the universe the result may be false and misleading.
3. **Lack Of Experts:**

As there are lack of experts to plan and conduct a sample survey, its execution and analysis, and its results would be Unsatisfactory and not trustworthy.

4. **Sometimes More Difficult Than Census Method:**

Sometimes the sampling plan may be complicated and requires more money, labor and time than a census method.

5. **Personal Bias:**

There may be personal biases and prejudices with regard to the choice of technique and drawing of sampling units.

6. **Choice Of Sample Size:**

If the size of the sample is not appropriate then it may lead to untrue characteristics of the population.

7. **Conditions Of Complete Coverage:**

If the information is required for each and every item of the universe, then a complete enumeration survey is better.

**Essentials of sampling:**

In order to reach a clear conclusion, the sampling should possess the following essentials:

1. **It must be representative:**

   The sample selected should possess the similar characteristics of the original universe from which it has been drawn.
2. **Homogeneity:**

Selected samples from the universe should have similar nature and should not have any difference when compared with the universe.

3. **Adequate samples:**

In order to have a more reliable and representative result, a good number of items are to be included in the sample.

4. **Optimization:**

All efforts should be made to get maximum results both in terms of cost as well as efficiency. If the size of the sample is larger, there is better efficiency and at the same time the cost is more. A proper size of sample is maintained in order to have optimized results in terms of cost and efficiency.

**STATISTICAL LAWS**

One of the basic reasons for undertaking a sample survey is to predict and generalize the results for the population as a whole. The logical process of drawing general conclusions from a study of representative items is called induction. In statistics, induction is a generalization of facts on the assumption that the results provided by an adequate sample may be taken as applicable to the whole. The fact that the characteristics of the sample provide a fairly good idea about the population characteristics is borne out by the theory of probability. Sampling is based on two fundamental principles of statistics theory viz, (i) the Law of Statistical Regularity and (ii) the Law of Inertia of Large Numbers.

**THE LAW OF STATISTICAL REGULARITY**

The Law of Statistical Regularity is derived from the mathematical theory of probability. According to W.I.King, “the Law of Statistical Regularity formulated in the mathematical theory of probability lays down that a moderately large number of items chosen at random from a very large group are almost sure to have the characteristics of the large group.” For example, if we want to find out the average income of 10,000 people,
we take a sample of 100 people and find the average. Suppose another person takes another sample of 100 people from the same population and finds the average, the average income found out by both the persons will have the least difference. On the other hand if the average income of the same 10,000 people is found out by the census method, the result will be more or less the same.

**Characteristics**

1. The item selected will represent the universe and the result is generalized to universe as a whole.
2. Since sample size is large, it is representative of the universe.
3. There is a very remote chance of bias.

**LAW OF INERTIA OF LARGE NUMBERS**

The Law of inertia of Large Numbers is an immediate deduction from the Principle of Statistical Regularity. Law of Inertia of Large Numbers states, “Other things being equal, as the sample size increases, the results tend to be more reliable and accurate.” This is based on the fact that the behavior or a phenomenon en masse. i.e., on a large scale is generally stable. It implies that the total change is likely to be very small, when a large number or items are taken in a sample. The law will be true on an average. If sufficient large samples are taken from the patent population, the reverse movements of different parts in the same will offset by the corresponding movements of some other parts.

**Sampling Errors:**

In a sample survey, since only a small portion of the population is studied its results are bound to differ from the census results and thus, have a certain amount of error. In statistics the word error is used to denote the difference between the true value and the estimated or approximated value. This error would always be there no matter that the sample is drawn at random and that it is highly representative. This error is attributed to fluctuations of sampling and is called sampling error. Sampling error exist due to the fact that only a sub set of the population has been used to estimate the population parameters and draw inferences about the population. Thus, sampling error is present only in a sample survey and is completely absent in census method.
Sampling errors occur primarily due to the following reasons:

1. **Faulty selection of the sample:**

   Some of the bias is introduced by the use of defective sampling technique for the selection of a sample e.g. Purposive or judgment sampling in which the investigator deliberately selects a representative sample to obtain certain results. This bias can be easily overcome by adopting the technique of simple random sampling.

2. **Substitution:**

   When difficulties arise in enumerating a particular sampling unit included in the random sample, the investigators usually substitute a convenient member of the population. This obviously leads to some bias since the characteristics possessed by the substituted unit will usually be different from those possessed by the unit originally included in the sample.

3. **Faulty demarcation of sampling units:**

   Bias due to defective demarcation of sampling units is particularly significant in area surveys such as agricultural experiments in the field of crop cutting surveys etc. In such surveys, while dealing with border line cases, it depends more or less on the discretion of the investigator whether to include them in the sample or not.

4. **Error due to bias in the estimation method:**

   Sampling method consists in estimating the parameters of the population by appropriate statistics computed from the sample. Improper choice of the estimation techniques might introduce the error.

5. **Variability of the population:**

   Sampling error also depends on the variability or heterogeneity of the population to be sampled.

**Sampling errors are of two types:** **Biased Errors and Unbiased Errors**
**Biased Errors:**

The errors that occur due to a bias of prejudice on the part of the informant or enumerator in selecting, estimating measuring instruments are called biased errors. Suppose for example, the enumerator uses the deliberate sampling method in the place of simple random sampling method, then it is called biased errors. These errors are cumulative in nature and increase when the sample size also increases. These errors arise due to defect in the methods of collection of data, defect in the method of organization of data and defect in the method of analysis of data.

**Unbiased Errors:**

Errors which occur in the normal course of investigation or enumeration on account of chance are called unbiased errors. They may arise accidentally without any bias or prejudice. These errors occur due to faulty planning of statistical investigation.

To avoid these errors, the statistician must take proper precaution and care in using the correct measuring instrument. He must see that the enumerators are also not biased. Unbiased errors can be removed with the proper planning of statistical investigations. Both these errors should be avoided by the statisticians.

**Reducing Sampling Errors:**

Errors in sampling can be reduced if the size of sample is increased. This is shown in the following diagram.

From the above diagram it is clear that when the size of the sample increases, sampling error decreases. And by this process samples can be made more representatives to the population.

**Testing of hypothesis:**

As a part of investigation, samples are drawn from the population and results are derived to help in taking the decisions. But such decisions involve an element of uncertainty causing wrong decisions. Hypothesis is an assumption which may or may not be true about a population parameter. For example, if we toss a coin 200 times, we may get 110 heads and 90 tails.
At this instance, we are interested in testing whether the coin is unbiased or not.

Therefore, we may conduct a test to judge the significance of the difference of sampling or otherwise. To carry out a test of significance, the following procedure has to be followed:

1. Framing the Hypothesis:

   To verify the assumption, which is based on sample study, we collect data and find out the difference between the sample value and the population value. If there is no difference found or the difference is very small then the hypothetical value is correct. Generally two hypotheses complementary to each other are constructed, and if one is found correct, the other is rejected.

   (a) Null Hypothesis:

   The random selection of the samples from the given population makes the tests of significance valid for us. For applying any test of significance we first set up a hypothesis- a definite statement about the population parameter/s. Such a statistical hypothesis, which is under test, is usually a hypothesis of no difference and hence is called null hypothesis. It is usually denoted by $H_0$. In the words of Prof. R.A.Fisher “Null Hypothesis is the hypothesis which is tested for possible rejection under the assumption that it is true.”

   (b) Alternative Hypothesis.

   Any hypothesis which is complementary to the null hypothesis is called an alternative hypothesis. It is usually denoted by $H_1$. It is very important to explicitly state the alternative hypothesis in respect of any null hypothesis $H_0$, because the acceptance or rejection of $H_0$ is meaningful only if it is being tested against an opposite hypothesis. For example, if we want to test the null hypothesis that the population has a specified mean $\mu_0$ (say), i.e., $H_0: \mu = \mu_0$ then the alternative hypothesis could be:

   (i) $H_1: \mu \neq \mu_0$ (i.e., $\mu > \mu_0$ or $\mu < \mu_0$)

   (ii) $H_1: \mu > \mu_0$ (iii) $H_1: \mu < \mu_0$
The alternative hypothesis (i) is known as a two-tailed alternative and the alternatives in (ii) and (iii) are known as right-tailed and left-tailed alternatives. Accordingly, the corresponding tests of significance are called two-tailed, right-tailed and left-tailed tests respectively.

The null hypothesis consists of only a single parameter value and is usually simple while alternative hypothesis is usually composite.

**Types Of Errors In Testing Of Hypothesis:**

As stated earlier, the inductive inference consists in arriving at a decision to accept or reject a null hypothesis (Ho) after inspecting only a sample from it. As such an element of risk – the risk of taking wrong decision is involved. In any test procedure, the four possible mutually disjoint and exhaustive decisions are:

i. **Reject Ho when actually it is not true** i.e., when Ho is false.
ii. **Accept Ho when it is true.**
iii. **Reject Ho when it is true.**
iv. **Accept Ho when it is false.**

The decisions in (i) and (ii) are correct decisions while the decisions in (iii) and (iv) are wrong decisions. These decisions may be expressed in the following dichotomous table:

<table>
<thead>
<tr>
<th>True State</th>
<th>Decision from sample</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Ho True</td>
<td>Reject Ho</td>
<td>Accept Ho</td>
</tr>
<tr>
<td>Ho True</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ho False</td>
<td>Correct</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Thus, in testing of hypothesis we are likely to commit two types of errors. The error of rejecting Ho when Ho is true is known as Type I Error and the error of accepting Ho when Ho is false is known as Type II Error.
For example, in the Industrial Quality Control, while inspecting the quality of a manufactured lot, the Inspector commits Type I Error when he rejects a good lot and he commits Type II Error when he accepts a bad lot.

**SUMMARY**

Nowadays questionnaire method of data collection has become very popular. It is a very powerful tool to collect required data in shortest period of time and with little expense. It is scientific too. But drafting of questionnaire is a very skilled and careful work. Therefore, there are certain requirements and essentials which should be followed at the time of framing the questionnaire. They include the following viz., (i) the size of the questionnaire should be small, (ii) questions should be very clear in understanding, (iii) questions should be put in a logical order, (iv) questions should have simple meaning etc. Apart from this, multiple choice questions should be asked. Questionnaire should be pre tested before going for final data collection. Information supplied should be cross checked for any false or insufficient information. After all these formalities have been completed, a covering note should accompany the questionnaire explaining various purposes, designs, units and incentives.

There are two ways of survey- census survey and Sample survey through which data can be collected. Census survey means total enumeration i.e., collecting data from each and every unit of the universe, whereas sample survey concentrates on collecting data from a few units of the universe selected scientifically for the purpose. Since census method is more time taking, expensive and labor intensive, it becomes impractical to depend on it. Therefore, sample survey is preferred which is scientific, less expensive, less time taking and less labor intensive too.

The merits and demerits of the methods are:

**Merits:**

It reduces cost, saves time and is more reliable. It provides more detailed information and is sometimes the only method to depend upon for administrative convenience and scientifically.
Demerits:

Sometimes samples may not be representative and may give illusory conclusions. There are lack of experts and sometimes it is more difficult than the census method, since there might arise personal bias and the determination of the size of the sample might be very difficult.

Apart from these, the essentials of sampling that are to be followed are: samples must be representative, samples must be homogeneous and the number of samples must be adequate. When a researcher resorts to sampling, he intends to collect some data which would help him to draw results and finally take a decision. When he takes a decision it’s on the basis of hypothesis which is precisely an assumption and is prone to two types of errors-Type I Error and Type II Error. When a researcher rejects a correct hypothesis, he commits type I error and when he accepts a wrong hypothesis he commits type II error. The researcher should try to avoid both types of errors but committing type II error is more harmful than type I error.

**SELF ASSESMENT QUESTIONS (SAQS)**

1. Explain questionnaire and examine its main characteristics.  
   (refer to the introduction part of the questionnaire section)
2. Explain main requirements of a good questionnaire.  
   (refer to the sub points from 1 to 11)
3. What is sampling? Explain its main merits and demerits.  
   (refer to the introduction and the following part of the lesson)
   (refer to the point framing the hypothesis)
5. What are type i error and type ii error?  
   (refer to types of error in hypothesis)

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Lesson Outline

- Procedures Adopted In Experiments
- Meaning Of Experiments
- Research Design In Case Of Hypothesis Testing Research Studies
- Basic Principles In Experimental Designs
- Prominent Experimental Designs

Learning Objectives

After reading this lesson you should be able to understand the

- Nature and meaning of experiments
- Kinds of experiments

Introduction

The meaning of experiment lies in the process of examining the truth of a statistical hypothesis related to some research problem. For example, a researcher can conduct an experiment to examine the newly developed medicine. Experiment is of two types: absolute experiment and comparative experiment. When a researcher wants to determine the impact of a fertilizer on the yield of a crop it is a case of absolute experiment. On the other hand, if he wants to determine the impact of one fertilizer as compared to the impact of some other fertilizer, the experiment will then be called as a comparative experiment. Normally, a researcher conducts a comparative experiment when he talks of designs of experiments.
Research design can be of three types:

1. Research design in the case of descriptive and diagnostic research studies,
2. Research design in the case of exploratory research studies, and
3. Research design in the case of hypothesis testing research studies.

Here we are mainly concerned with the third one which is Research design in the case of hypothesis testing research studies.

**Research design in the case of hypothesis testing research studies:**

Hypothesis testing research studies are generally known as experimental studies. This is a study where a researcher tests the hypothesis of causal relationships between variables. This type of study requires some procedures which will not only reduce bias and increase reliability, but will also permit drawing inferences about causality. Most of the times, experiments meet these requirements. Prof. Fisher is considered as the pioneer of this type of studies (experimental studies). He did pioneering work when he was working at Rothamsted Experimental Station in England which was a centre for Agricultural Research. While working there, Prof. Fisher found that by dividing plots into different blocks and then by conducting experiments in each of these blocks whatever information that were collected and inferences drawn from them happened to be more reliable. This was where he was inspired to develop certain experimental designs for testing hypotheses concerning scientific investigations. Nowadays, the experimental design is used in researches relating to almost every discipline of knowledge. Prof. Fisher laid three principles of experimental designs:

1. The Principle of Replication
2. The Principle of Randomization and
3. The Principle of Local Cont

1. **The Principle Of Replication:**

According to this principle, the experiment should be repeated more than once. Thus, each treatment is applied in many experimental
units instead of one. This way the statistical accuracy of the experiments is increased. For example, suppose we are going to examine the effect of two varieties of wheat. Accordingly, we divide the field into two parts and grow one variety in one part and the other variety in the other. Then we compare the yield of the two parts and draw conclusion on that basis. But if we are to apply the principle of replication to this experiment, then we first divide the field into several parts, grow one variety in half of these parts and the other variety in the remaining parts. Then we collect the data of yield of the two varieties and draw conclusion by comparing the same. The result so obtained will be more reliable in comparison to the conclusion we draw without applying the principle of replication. The entire experiment can be repeated several times for better results.

2. **The Principle Of Randomization:**

When we conduct an experiment, the principle of randomization provides us a protection against the effects of extraneous factors. This means that this principle indicates that the researcher should design or plan the experiment in such a way that the variations caused by extraneous factors can all be combined under the general heading of ‘chance’. For example, when a researcher grows one variety of wheat, say, in the first half of the parts of a field and the other variety he grows in the other half, then it is just possible that the soil fertility may be different in the first half in comparison to the other half. If this is so the researcher’s result is not realistic. In this situation, he may assign the variety of wheat to be grown in different parts of the field on the basis of some random sampling technique i.e., he may apply randomization principle and protect himself against the effects of the extraneous factors. Therefore, by using the principle of randomization, he can draw a better estimate of the experimental error.

3. **The Principle Of Local Control:**

This is another important principle of experimental designs. Under this principle, the extraneous factor which is the known source of variability is made to vary deliberately over as wide a range as necessary. This needs to be done in such a way that the variability it causes can be measured and hence eliminated from the experimental error. The experiment should be planned in such a way that the researcher can perform a two-way analysis of variance, in which the total variability of the data is divided into three
components attributed to treatments (varieties of wheat in this case), the extraneous factor (soil fertility in this case) and experimental error. In short, through the principle of local control we can eliminate the variability due to extraneous factors from the experimental error.

Kinds Of Experimental Designs And Control

Experimental designs refer to the framework of structure of an experiment and as such there are several experimental designs. Generally, experimental designs are classified into two broad categories: informal experimental designs and formal experimental designs. Informal experimental designs are those designs that normally use a less sophisticated form of analysis based on differences in the magnitudes, whereas formal experimental designs offer relatively more control and use precise statistical procedures for analysis. Important experimental designs are discussed below:

1. Informal Experimental Designs:

   (i) Before and after without control design
   (ii) After only with control design
   (iii) Before and after with control design

2. Formal Experimental Designs:

   (i) Completely randomized design (generally called C.R design)
   (ii) Randomized block design (generally called R.B design)
   (iii) Latin square design (generally called L.S design)
   (iv) Factorial designs.

(i) Before and after without control design:

In this design, a single test group or area is selected and the dependent variable is measured before introduction of the treatment. Then the treatment is introduced and the dependent variable is measured again after the treatment has been introduced. The effect of the treatment would be equal to the level of the phenomenon after the treatment minus the level of the phenomenon before the treatment. Thus, the design can be presented in the following manner:
The main difficulty of such a design is that with the passage of time considerable extraneous variations may be there in its treatment effect.

(ii) **After-only with control design:**

Two groups or areas are selected in this design and the treatment is introduced into the test area only. Then the dependent variable is measured in both the areas at the same time. Treatment impact is assessed by subtracting the value of the dependent variable in the control area from its value in the test area. The design can be presented in the following manner:

<table>
<thead>
<tr>
<th>Test area: Treatment introduced</th>
<th>Level of phenomenon after Treatment (Y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Control area:</td>
<td>Level of phenomenon Without treatment (Z)</td>
</tr>
</tbody>
</table>

Treatment Effect = (Y)-(Z)

The basic assumption in this type of design is that the two areas are identical with respect to their behavior towards the phenomenon considered. If this assumption is not true, there is the possibility of extraneous variation entering into the treatment effect.

(iii) **Before and after with control design:**

In this design, two areas are selected and the dependent variable is measured in both the areas for an identical time-period before the treatment. Thereafter, the treatment is introduced into the test area only
and the dependent variable is measured in both for and identical time period after the introduction of the treatment. The effect of the treatment is determined by subtracting the change in the dependent variable in the control area from the change in the dependent variable in test area. This design can be shown in the following way:

<table>
<thead>
<tr>
<th>Time Period I</th>
<th>Time Period II</th>
</tr>
</thead>
<tbody>
<tr>
<td>Test area:</td>
<td>Level of phenomenon</td>
</tr>
<tr>
<td>Before treatment (X)</td>
<td>Introduced</td>
</tr>
<tr>
<td>Control area:</td>
<td>Level of phenomenon</td>
</tr>
<tr>
<td>Without treatment</td>
<td>(A)</td>
</tr>
</tbody>
</table>

Treatment Effect = (Y-X)-(Z-A)

This design is superior to the previous two designs because it avoids extraneous variation resulting both from the passage of time and from non-comparability of the rest and control areas. But at times, due to lack of historical data time or a comparable control area, we should prefer to select one of the first two informal designs stated above.

2. Formal Experimental Design

(i) Completely randomized design:

This design involves only two principles i.e., the principle of replication and the principle of randomization of experimental designs. Among all other designs this is the simpler and easier because it's procedure and analysis are simple. The important characteristic of this design is that the subjects are randomly assigned to experimental treatments. For example, if the researcher has 20 subjects and if he wishes to test 10 under treatment A and 10 under treatment B, the randomization process gives every possible group of 10 subjects selected from a set of 20 an equal opportunity of being assigned to treatment A and treatment B. One way analysis of variance (one way ANOVA) is used to analyze such a design.
(ii) **Randomized block design:**

R. B. Design is an improvement over the C.R. design. In the R .B. Design, the principle of local control can be applied along with the other two principles of experimental designs. In the R.B. design, subjects are first divided into groups, known as blocks, such that within each group the subjects are relatively homogenous in respect to some selected variable. The number of subjects in a given block would be randomly assigned to each treatment. Blocks are the levels at which we hold the extraneous factor fixed, so that its contribution to the total variability of data can be measured. The main feature of the R.B. design is that, in this, each treatment appears the same number of times in each block. This design is analyzed by the two-way analysis of variance (two-way ANOVA) technique.

(iii) **Latin squares design:**

The Latin squares design (L.S design) is an experimental design which is very frequently used in agricultural research. Since agriculture depends upon nature to a large extent, the condition of research and investigation in agriculture is different than the other studies. For example, an experiment has to be made through which the effects of fertilizers on the yield of a certain crop, say wheat, are to be judged. In this situation, the varying fertility of the soil in different blocks in which the experiment has to be performed must be taken into consideration; otherwise the results obtained may not be very dependable because the output happens to be the effects of not only of fertilizers, but also of the effect of fertility of soil. Similarly there may be the impact of varying seeds of the yield. In order to overcome such difficulties, the L.S. design is used when there are two major extraneous factors such as the varying soil fertility and varying seeds. The Latin square design is such that each fertilizer will appear five times but will be used only once in each row and in each column of the design. In other words, in this design, the treatment is so allocated among the plots that no treatment occurs more than once in any one row or any one column. This experiment can be shown with the help of the following diagram:
From the above diagram, it is clear that in L.S. design the field is divided into as many blocks as there are varieties of fertilizers. Then, each block is again divided into as many parts as there are varieties of fertilizers in such a way that each of the fertilizer variety is used in each of the block only once. The analysis of L.S. design is very similar to the two-way ANOVA technique.

(iv) **Factorial design:**

Factorial designs are used in experiments where the effects of varying more than one factor are to be determined. These designs are used more in economic and social matters where usually a large number of factors affect a particular problem. Factorial designs are usually of two types:

(i) Simple factorial designs and  
(ii) Complex factorial designs.

(i) **Simple factorial design:**

In simple factorial design, the effects of varying two factors on the dependent variable are considered but when an experiment is done with more than two factors, complex factorial designs are used. Simple factorial design is also termed as a 'two-factor-factorial design,' whereas complex factorial design is known as 'multi-factor-factorial design.'
(ii) **Complex factorial designs:**

When the experiments with more than two factors at a time are conducted, it involves the use of complex factorial designs. A design which considers three or more independent variables simultaneously is called a complex factorial design. In case of three factors with one experimental variable, two treatments and two levels, complex factorial design will contain a total of eight cells. This can be seen through the following diagram:

2x2x2 COMPLEX FACTORIAL DESIGN

<table>
<thead>
<tr>
<th>Experimental variable</th>
<th>Treatment A</th>
<th>Treatment B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Control variable 2</td>
<td>Control variable 2</td>
<td>Control variable 2</td>
</tr>
<tr>
<td>Level I</td>
<td>Level II</td>
<td>Level III</td>
</tr>
<tr>
<td>Cell 1</td>
<td>Cell 3</td>
<td>Cell 5</td>
</tr>
<tr>
<td>Cell 2</td>
<td>Cell 4</td>
<td>Cell 6</td>
</tr>
</tbody>
</table>

A pictorial presentation is given of the design shown above in the following:
The dotted line cell in this diagram corresponds to cell I of the above stated 2x2x2 design and is for treatment A, level I of the control variable 1, and level II of the control variable 2. From this design, it is possible to determine the main effects for three variables i.e., one experimental and true control variables. The researcher can also determine the interaction between each possible pair of variables (such interactions are called ‘first order interactions’) and interaction between variable taken in triplets (such interactions are called second order interactions). In case of a 2x2x2 design, the further given first order interactions are possible:

- Experimental variable with control variable 1  (or EV x CV 1);
- Experimental variable with control variable 2  (or EV x CV 2);
- Control variable 1 with control variable 2    (or CV 1 x CV 2);

There will be one second order interaction as well in the given design (it is between all the three variables i.e., EV x CV 1 x CV 2).

To determine the main effect for the experimental variable, the researcher must necessarily compare the combined mean of data in cells 1, 2, 3 and 4 for Treatment A with the combined mean of data in cells 5, 6, 7 and 8 for Treatment B. In this way the main effect of experimental variable, independent of control variable 1 and variable 2, is obtained. Similarly, the main effect for control variable 1, independent experimental variable and control variable 2, is obtained if we compare the combined mean of data in cells 1, 3, 5 and 7 with the combined mean of data in cells 2, 4, 6 and 8 of our 2x2x2 factorial design. On similar lines, one can determine the effect of control variable 2 independent of experimental variable and control variable 1, if the combined mean of data in cells 1, 2, 5 and 6 are compared with the combined mean of data in cells 3, 4, 7 and 8.

To obtain the first order interaction, say, for EV x CV 1 in the above stated design, the researcher must necessarily ignore control variable 2 for which purpose he may develop 2x2 design from the 2x2x2 design by combining the data of the relevant cells of the latter design as has been shown on next page:
Similarly, the researcher can determine other first order interactions. The analysis of the first order interaction in the manner described above is essentially a simple factorial analysis as only two variables are considered at a time and the remaining ones are ignored. But the analysis of the second order interaction would not ignore one of the three independent variables in case of a 2x2x2 design. The analysis would be termed as a complex factorial analysis.

It may, however, be remembered that the complex factorial design need not necessarily be of 2x2x2 type design, but can be generalized to any number and combinations of experimental and control independent variables. Of course, the greater the number of independent variables included in a complex factorial design, the higher the order of the interaction analysis possible. But the overall task goes on becoming more and more complicated with the inclusion of more and more independent variables in our design.

Factorial designs are used mainly because of the two advantages -

(i) They provide equivalent accuracy (as happens in the case of experiments with only one factor) with less labour and as such are source of economy. Using factorial designs, we can determine the effects of two (in simple factorial design) or more (in case of complex factorial design) factors (or variables) in one single experiment. (ii) they permit various other comparisons of interest. For example, they give information about such effects which cannot be obtained by treating one single factor at a time. The determination of interaction effects is possible in case of factorial designs.
Conclusion

There are several research designs and the researcher must decide in advance of collection and analysis of data as to which design would prove to be more appropriate for his research project. He must give due weight to various points such as type of universe and its nature, the objective of the study, the source list or the sampling frame, desired standard accuracy and the like when taking a decision in respect of the design for his research project.

SUMMARY

Experiment is the process of examining the truth of a statistical hypothesis related to some research problem. There are two types of experiments - absolute and comparative. There are three types of research designs - research design for descriptive and diagnostic research, research design for exploratory research studies and research design for hypothesis testing. Prof. Fisher has laid three principles of experimental design. They are Principle of Replication, Principle of Randomization and Principle of Local Control. There are different kinds of experimental designs. Some of them are Informal experimental design, After only with control design, Formal experimental design, Completely randomized design, Randomized block design, Latin square design and Factorial design.

SELF ASSESMENT QUESTIONS (SAQS)

1. Explain the meaning and types of experiment (Ref. Introduction and types of research design next to introduction)

2. Explain informal designs. (Ref. i,ii,iii in informal experiment design portion.)

3. Explain formal experimental design and control. (Ref. i,ii,iii,iv in formal experiment design section)

4. Explain complex factorial design.

***
UNIT II

Lesson 4. Observation

Lesson Outline

- Steps In Observation
- Meaning And Characteristics Of Observation
- Types Of Observation
- Stages Of Observation
- Problems, Merits And Demerits

Lesson Objectives

After reading this lesson you will be able to know

- Meaning and types of observation
- Stages through which observation passes
- Steps followed and the problems coming in observation
- Merits and demerits

Introduction

Observation is a method that employs vision as its main means of data collection. It implies the use of eyes rather than of ears and the voice. It is accurate watching and noting of phenomena as they occur with regard to the cause and effect or mutual relations. It is watching other persons’ behavior as it actually happens without controlling it. For example, watching bonded labourer’s life, or treatment of widows and their drudgery at home, provide graphic description of their social life and sufferings. Observation is also defined as “a planned methodical watching that involves constraints to improve accuracy”.

89
CHARACTERISTICS OF OBSERVATION

Scientific observation differs from other methods of data collection specifically in four ways: (i) observation is always direct while other methods could be direct or indirect; (ii) field observation takes place in a natural setting; (iii) observation tends to be less structured; and (iv) it makes only the qualitative (and not the quantitative) study which aims at discovering subjects’ experiences and how subjects make sense of them (phenomenology) or how subjects understand their life (interpretivism).

Loftland (1955) has said that this method is more appropriate for studying lifestyles or sub-cultures, practices, episodes, encounters, relationships, groups, organizations, settlements and roles etc. Black and Champion (1976) have given the following characteristics of observation:

- Behavior is observed in natural surroundings.
- It enables understanding significant events affecting social relations of the participants.
- It determines reality from the perspective of observed person himself.
- It identifies regularities and recurrences in social life by comparing data in our study with that of other studies.

Besides, four other characteristics are:

- Observation involves some control pertaining to the observation and to the means he uses to record data. However, such controls do not exist for the setting or the subject population.
- It is focused on hypotheses-free inquiry.
- It avoids manipulations in the independent variable i.e., one that is supposed to cause other variable(s) and is not caused by them.
- Recording is not selective.

Since at times, observation technique is indistinguishable from experiment technique, it is necessary to distinguish the two.
Observation involves few controls than the experiment technique.

1. The behaviour observed in observation is natural, whereas in experiment it is not always so.
2. The behavior observed in experiment is more molecular (of a smaller unit), while one in observation is molar.
3. In observation, fewer subjects are watched for long periods of time in more varied circumstances than in experiment.
4. Training required in observation study is directed more towards sensitizing the observer to the flow of events, whereas training in experiments serves to sharpen the judgment of the subject.
5. In observational study, the behavior observed is more diffused. Observational methods differ from one another along several variables or dimensions.

**Merits:**

1. We get original data
2. We get more accurate and reliable data.
3. Satisfactory information can be extracted by the investigator through indirect questions.
4. Data are homogeneous and comparable.
5. Additional information can be gathered.
6. Misinterpretation of questions can be avoided.

**Demerits:**

1. It is time consuming and costs more.

**SELF ASSESSMENT QUESTIONS (SAQS)**

1. What are the characteristics of observation?
2. How do you differentiate observation from experiment?
UNIT – III

Statistical Analysis

CONTENTS

1. Probability
2. Probability Distribution
   2.1 Binomial Distribution
   2.2 Poisson Distribution
   2.3 Normal Distribution
3. Testing of Hypothesis
   3.1 Small Sample
   3.2 Large Sample Test
4. Χ² Test

Objectives:

The objectives of the present chapter are:

- To examine the utility of various statistical tools in decision making.
- To inquire about the testing of a hypothesis

1. PROBABILITY

If an experiment is repeated under essentially homogeneous and similar conditions, two possible conclusions can be arrived. They are: the results are unique and the outcome can be predictable and result is not unique but may be one of the several possible outcomes. In this context, it is better to understand various terms pertaining to probability before examining the probability theory. The main terms are explained as follows:

(i) Random Experiment

An experiment which can be repeated under the same conditions and the outcome cannot be predicted under any circumstances is known
as random experiment. For example: An unbiased coin is tossed. Here we are not in a position to predict whether head or tail is going to occur. Hence, this type of experiment is known as random experiment.

(ii) Sample Space

A set of possible outcomes of a random experiment is known as sample space. For example, in the case of tossing of an unbiased coin twice, the possible outcomes are HH, HT, TH and TT. This can be represented in a sample space as $S = \{HH, HT, TH, TT\}$.

(iii) An Event

Any possible outcomes of an experiment are known as an event. In the case of tossing of an unbiased coin twice, HH is an event. An event can be classified into two. They are: (a) Simple events, and (ii) Compound events. Simple event is an event which has only one sample point in the sample space. Compound event is an event which has more than one sample point in the sample space. In the case of tossing of an unbiased coin twice HH is a simple event and TH and TT are the compound events.

(iv) Complementary Event

$A$ and $A'$ are the complementary event if $A'$ consists of all those sample point which is not included in $A$. For instance, an unbiased dice is thrown once. The probability of an odd number turns up are complementary to an even number turns up. Here, it is worth mentioning that the probability of sample space is always is equal to one. Hence, the $P(A') = 1 - P(A)$.

(v) Mutually Exclusive Events

$A$ and $B$ are the two mutually exclusive events if the occurrence of $A$ precludes the occurrence of $B$. For example, in the case of tossing of an unbiased coin once, the occurrence of head precludes the occurrence of tail. Hence, head and tail are the mutually exclusive event in the case of tossing of an unbiased coin once. If $A$ and $B$ are mutually exclusive events, then the probability of occurrence of $A$ or $B$ is equal to sum of their individual probabilities. Symbolically, it can be presented as:
\[ P(A \cup B) = P(A) + P(B) \]

If \(A\) and \(B\) is joint sets, then the addition theorem of probability can be stated as:

\[ P(A \cup B) = P(A) + P(B) - P(AB) \]

(vi) **Independent Event**

\(A\) and \(B\) are the two independent event if the occurrence of \(A\) does not influence the occurrence of \(B\). In the case of tossing of an unbiased coin twice, the occurrence of head in the first toss does not influence the occurrence of head or tail in the toss. Hence, these two events are called independent events. In the case of independent event, the multiplication theorem can be stated as the probability of \(A\) and \(B\) is the product of their individual probabilities. Symbolically, it can be presented as:-

\[ P(A \cap B) = P(A) \cdot P(B) \]

**Addition Theorem of Probability**

Let \(A\) and \(B\) be the two mutually exclusive events, then the probability of \(A\) or \(B\) is equal to the sum of their individual probabilities. (for detail refer mutually exclusive events)

**Multiplication Theorem of Probability**

Let \(A\) and \(B\) be the two independent events, then the probability of \(A\) and \(B\) is equal to the product of their individual probabilities. (for details refer independent events)

**Example:** The odds that person \(X\) speaks the truth are 4:1 and the odds that \(Y\) speaks the truth are 3:1. Find the probability that:

i. Both of them speak the truth,
ii. Any one of them speak the truth and
iii. Truth may not be told.
Solution:

The probability of X speaks the truth  = 1/5
The probability that X speaks lie = 4/5
The probability that Y speaks the truth = 1/4
The probability that Y speaks lie = ¼

(i) Both of them speak truth = P(X) * P(Y)
   = 1/5 * 1/4
   = 1/20 (independent event)

(ii) Any one of them speak truth = P(X) + P(Y) - P(X*Y)
   = 1/5 + 1/4 - 1/5*1/4
   = 8/20
   = 2/5 (not mutually exclusive events)

(iii) Truth may not be told
     = 1 – p(any one of them speak truth)(complementary event)
     = 1 – 2/5
     = 3/5.

2. PROBABILITY DISTRIBUTION

If X is discrete random variable which takes the values of \(x_1, x_2, x_3, \ldots, X_n\) and the corresponding probabilities are \(p_1, p_2, \ldots, p_n\), then, X follows the probability distribution. The two main properties of probability distribution are: (i) \(P(X_i)\) is always greater than or equal to zero and less than or equal to one, and (ii) the summation of probability distribution is always equal to one. For example, tossing of an unbiased coin twice.

Then the probability distribution is:

<table>
<thead>
<tr>
<th>X (probability of obtaining head):</th>
<th>0</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>P(xi)</td>
<td>¼</td>
<td>½</td>
<td>¼</td>
</tr>
</tbody>
</table>

Expectation of probability

Let X be the discrete random variable which takes the value of \(x_1, x_2, \ldots, x_n\) then the respective probability is \(p_1, p_2, \ldots, p_n\), then the
expectation of probability distribution is \( p_1x_1 + p_2x_2 + \ldots + p_nx_n \).
In the above example, the expectation of probability distribution is \( (0 \times \frac{1}{4} + 1 \times \frac{1}{2} + 2 \times \frac{1}{4}) = 1 \).

### 2.1 BINOMIAL DISTRIBUTION

The binomial distribution also known as ‘Bernoulli Distribution’ is associated with the name of a Swiss mathematician, James Bernoulli who is also known as Jacques or Jakon (1654 – 1705). Binomial distribution is a probability distribution expressing the probability of one set of dichotomous alternatives. It can be explained as follows:

i. If an experiment is repeated under the same conditions for a fixed number of trials, say, \( n \).

ii. In each trial, there are only two possible outcomes of the experiment. Let us define it as “success” or “failure”. Then the sample space of possible outcomes of each experiment is:

iii. \( S = \{\text{failure, success}\} \)

iv. The probability of a success denoted by \( p \) remains constant from trial to trial and the probability of a failure denoted by \( q \) which is equal to \( 1 - p \).

v. The trials are independent in nature i.e., the outcomes of any trial or sequence of trials do not affect the outcomes of subsequent trials. Hence, the multiplication theorem of probability can be applied for the occurrence of success and failure. Thus, the probability of success or failure is \( p \cdot q \).

vi. Let us assume that we conduct an experiment in \( n \) times. Out of which \( x \) times be the success and failure is \( (n-x) \) times. The occurrence of success or failure in successive trials is mutually exclusive events. Hence, we can apply addition theorem of probability.

vii. Based on the above two theorems, the probability of success or failure is

\[
P(X) = \binom{n}{x} p^x q^{n-x}
\]
\[ \frac{n!}{x!(n-x)!} \cdot p^x q^{n-x} \]

where \( p \) = probability of success in a single trial, \( q = 1 - p \), \( n = \) Number of trials and \( x = \) no. of successes in \( n \) trials.

Thus, for an event \( A \) with probability of occurrence \( p \) and non-occurrence \( q \), if \( n \) trials are made, probability distribution of the number of occurrences of \( A \) will be as set. If we want to obtain the probable frequencies of the various outcomes in \( n \) sets of \( N \) trials, the following expression shall be used: \( N(p + q)^n \)

\[
N(p + q)^n = Np^n + ^nC_1p^{n-1}q + ^nC_2p^{n-2}q^2 + \ldots + ^nC_r p^{n-r}q^r + \ldots q^n.
\]

The frequencies obtained by the above expansion are known as expected or theoretical frequencies. On the other hand, the frequencies actually obtained by making experiments are called actual or observed frequencies. Generally, there is some difference between the observed and expected frequencies but the difference becomes smaller and smaller as \( N \) increases.

**Obtaining Coefficient Of The Binomial Distribution:**

The following rules may be considered for obtaining coefficients from the binomial expansion:

i. The first term is \( q^n \).

ii. The second term is \( ^nC_1q^{n-1}p \),

iii. In each succeeding term the power of \( q \) is reduced by 1 and the power of \( p \) is increased by 1.

iv. The coefficient of any term is found by multiplying the coefficient of the preceding term by the power of \( q \) in that preceding term, and dividing the products so obtained by one more than the power of \( p \) in that proceeding term.
Thus, when we expand \((q + p)^n\), we will obtain the following:

\[
(p + q)^n = p^n + \binom{n}{1}p^{n-1}q + \binom{n}{2}p^{n-2}q^2 + \ldots + \binom{n}{r}p^{n-r}q^r + \ldots + q^n.
\]

Where, 1, \(\binom{n}{1}, \binom{n}{2} \ldots \ldots\) are called the binomial coefficient. Thus in the expansion of \((p + q)^4\) we will have \((p + q)^4 = p^4 + 4p^3q + 6p^2q^2 + 4pq^3 + q^4\) and the coefficients will be 1, 4, 6, 4, 1.

From the above binomial expansion, the following general relationships should be noted:

i. The number of terms in a binomial expansion is always \(n + 1\),

ii. The exponents of \(p\) and \(q\), for any single term, when added together, always sum to \(n\).

iii. The exponents of \(p\) are \(n, (n - 1), (n - 2), \ldots \ldots, 1, 0\), respectively and the exponents of \(q\) are \(0, 1, 2, \ldots \ldots, (n - 1), n\), respectively.

iv. The coefficients for the \(n + 1\) terms of the distribution are always symmetrical in nature.

**Properties Of Binomial Distribution**

The main properties of binomial distribution are:-

i. The shape and location of binomial distribution changes as \(p\) changes for a given \(n\) or as \(n\) changes for a given \(p\). As \(p\) increases for a fixed \(n\), the binomial distribution shifts to the right.

ii. The mode of the binomial distribution is equal to the value of \(x\) which has the largest probability. The mean and mode are equal if \(np\) is an integer.

iii. As \(n\) increases for a fixed \(p\), the binomial distribution moves to the right, flattens and spreads out.

iv. The mean of the binomial distribution is \(np\) and it increases as \(n\) increases with \(p\) held constant. For larger \(n\) there are more possible outcomes of a binomial experiment and the probability associated with any particular outcome becomes smaller.
v. If n is larger and if neither p nor q is too close to zero, the binomial distribution can be closely approximated by a normal distribution with standardized variable given by 

\[ z = \frac{X - np}{\sqrt{npq}} \]

vi. The various constants of binomial distribution are:

- **Mean** = np
- **Standard Deviation** = \( \sqrt{npq} \)
- \( \mu_1 \) = 0
- \( \mu_2 \) = npq
- \( \mu_3 \) = npq(q - p)
- \( \mu_4 \) = 3np^2q^2 + npq(1 - 6pq).

**Skewness** = \( \frac{(q - p)^2}{npq} \)

**Kurtosis** = \( 3 + \frac{1 - 6pq}{npq} \)

**Illustrations:**

A coin is tossed four times. What is the probability of obtaining two or more heads?

**Solution:**

when a coin is tossed the probabilities of head and tail in case of an unbiased coin are equal, i.e., \( p = q = \frac{1}{2} \)

The various possibilities for all the events are the terms of the expansion \((q+p)^4\)

\[ (p + q)^4 = p^4 + 4p^3q + 6p^2q^2 + 4pq^3 + q^4 \]

Therefore, the probability of obtaining 2 heads is

\[ 6p^2q^2 = 6 \times \left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^2 = 3/8 \]
The probability of obtaining 3 heads is \(6p^3q^1 = 4 \times (\frac{1}{2})^3(\frac{1}{2})1\)
\[= \frac{1}{4}\]

The probability of obtaining 4 heads is \((q)^4 = (\frac{1}{2})^4\)
\[= \frac{1}{16}\]

Therefore, the probability of obtaining 2 or more heads is
\[
\begin{array}{cccc}
3 & 1 & 1 & 11 \\
8 & 4 & 16 & 16
\end{array}
\]

Illustration:

Assuming that half the population is vegetarian so that the chance of an individual being a vegetarian is \(\frac{1}{2}\) and assuming that 100 investigations can take sample of 10 individuals to verify whether they are vegetarians, how many investigation would you expect to report that three people or less were vegetarians?

Solution:

\(n=10\), \(p\), i.e., probability of an individual being vegetarian = \(\frac{1}{2}\).\(q = 1 - p = \frac{1}{2}\)

Using binomial distribution, we have \(P(r) = \binom{n}{r} q^{n-r}p^r\)

Putting the various values, we have

\[
10c_r \left(\frac{1}{2}\right)^{10-r} = 10cr = \left(\frac{1}{2}\right)^{10} = \frac{1}{1024}
\]

The probability that in a sample of 10, three or less people are vegetarian shall be given by:

\[
P(0) + p(1) + p(2) + p(3)
\]

\[
= \frac{1}{1024} \left[10c_0 + 10c_1 + 10c_2 + 10c_3\right]
\]

101
Hence out of 1000 investigators, the number of investigators who will
11
Report 3 or less vegetarians in a sample of 10 is \(1000 \times \frac{11}{64} = 172\).

2.2 POISSON DISTRIBUTION

Poisson distribution was derived in 1837 by a French Mathematician
Simeon D Poisson (1731 – 1840). In binomial distribution, the values of p
and q and n are given. There is a certainty of the total number of events.
But there are cases where p is very small and n is very large and such
case is normally related to poisson distribution. For example, persons
killed in road accidents, the number of defective articles produced by a
quality machine. Poisson distribution may be obtained as a limiting case
of binomial probability distribution, under the following condition.

i. \(P\), successes, approach zero \((p \rightarrow 0)\)
ii. \(np = m\) is finite.

The poisson distribution of the probabilities of occurrence of various rare
events (successes) 0,1,2,…. Are given below:

<table>
<thead>
<tr>
<th>Number of Success ((X))</th>
<th>Probabilities (p(X))</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>(e^{-m})</td>
</tr>
<tr>
<td>1</td>
<td>(me^{-m}/1!)</td>
</tr>
<tr>
<td>2</td>
<td>(m^2e^{-m}/2!)</td>
</tr>
<tr>
<td>r</td>
<td>(m^re^{-m}/r!)</td>
</tr>
<tr>
<td>n</td>
<td>(m^ne^{-m}/n!)</td>
</tr>
</tbody>
</table>

Where, \(e = 2.718\), and \(m\) = average number of occurrence of given
distribution.
The Poisson distribution is a discrete distribution with a parameter m. The various constants are:

i. Mean \( = m = p \)
ii. Standard Deviation \( = \sqrt{m} \)
iii. Skewness \( \beta_1 = \frac{1}{m} \)
iv. Kurtosis, \( \beta_2 = 3 + \frac{1}{m} \)
v. Variance \( = m \)

Illustration:

A book contains 100 misprints distributed randomly throughout its 100 pages. What is the probability that a page observed at random contains at least two misprints? Assume Poisson Distribution.

Solution:

\[
M = \frac{\text{Total number of misprints}}{\text{Total number of pages}} = \frac{100}{100} = 1
\]

Probability that a page contains at least two misprints:

\[
P(r \geq 2) = 1 - [P(0) + P(1)]
\]

\[
P(r) = \frac{m^r e^{-m}}{r!}
\]

\[
P(0) = \frac{1^0 e^{-1}}{0!} = \frac{1}{e} = \frac{1}{2.7183} = 0.368
\]

\[
P(1) = \frac{1^1 e^{-1}}{1!} = e^{-1} = \frac{1}{2.7183} = 0.368
\]

\[
P(r \geq 2) = 1 - [0.368 + 0.368] = 1 - 0.736 = 0.264
\]
\[
P(0) + p(1) = \frac{1}{2.718} + \frac{1}{2.718} = 0.736
\]

\[
P(r \geq 2) = 1 - [p(0) + p(1)] = 1 - 0.736 = 0.264
\]

**Illustration:**

If the mean of a Poisson distribution is 16, find (1) S.D. (2) \(B_1\) (3) \(B_2\) (4) \(\mu_3\) (5) \(\mu_4\)

**Solution:**

\[
m = 16
\]

1. S.D. \(= \sqrt{m} = \sqrt{16} = 4\)
2. \(B_1 = 1/m = 1/16 = 0.625\)
3. \(B_2 = 3 + 1/m = 3 + 0.625 = 3.0625\)
4. \(\mu_3 = m = 16\)
5. \(\mu_4 = m + 3m^2 = 16 + 3(16)^2 = 784\)

**2.3 NORMAL DISTRIBUTION**

The normal distribution was first described by Abraham Demoivre (1667-1754) as the limiting form of binomial model in 1733. Normal distribution was rediscovered by Gauss in 1809 and by Laplace in 1812. Both Gauss and Laplace were led to the distribution by their work on the theory of errors of observations arising in physical measuring processes particularly in astronomy.

The probability function of a Normal Distribution is defined as:

\[
P(X) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x - \mu)^2}{2\sigma^2}}
\]
Where, \( X \) = Values of the continuous random variable, \( \mu \) = Mean of the normal random variable, \( e = 2.7183, \pi = 3.1416 \)

**Relation Between Binomial, Poisson And Normal Distributions**

Binomial, Poisson and Normal Distribution are closely related to one other. When \( N \) is large while the probability \( P \) of the occurrence of an event is close to zero so that \( q = 1-p \) the binomial distribution is very closely approximated by the Poisson distribution with \( m = np \).

The Poisson distribution approaches a normal distribution with standardized variable \( (x – m)/\sqrt{m} \) as \( m \) increases to infinity.

**Normal distribution and its properties**

The important properties of the normal distribution are:-

1. The normal curve is “bell shaped” and symmetrical in nature. The distribution of the frequencies on either side of the maximum ordinate of the curve is similar with each other.

2. The maximum ordinate of the normal curve is at \( x = \mu \). Hence the mean, median and mode of the normal distribution coincide.

3. It ranges between - \( \infty \) to + \( \infty \)

4. The value of the maximum ordinate is \( 1/\sigma\sqrt{2\pi} \).

5. The points where the curve change from convex to concave or vice versa is at \( X = \mu \pm \sigma \).

6. The first and third quartiles are equidistant from median.

7. The area under the normal curve distribution are:

   a. \( \mu \pm 1\sigma \) covers 68.27% area;
   b. \( \mu \pm 2\sigma \) covers 95.45% area.
   c. \( \mu \pm 3\sigma \) covers 99.73% area.
8. When $\mu = 0$ and $\sigma = 1$, then the normal distribution will be a standard normal curve. The probability function of standard normal curve is

$$P(X) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}$$

The following table gives the area under the normal probability curve for some important value of $Z$.

<table>
<thead>
<tr>
<th>Distance from the mean ordinate in Terms of $\pm \sigma$</th>
<th>Area under the curve</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Z = \pm 0.6745$</td>
<td>0.50</td>
</tr>
<tr>
<td>$Z = \pm 1.0$</td>
<td>0.6826</td>
</tr>
<tr>
<td>$Z = \pm 1.96$</td>
<td>0.95</td>
</tr>
<tr>
<td>$Z = \pm 2.00$</td>
<td>0.9544</td>
</tr>
<tr>
<td>$Z = \pm 2.58$</td>
<td>0.99</td>
</tr>
<tr>
<td>$Z = \pm 3.0$</td>
<td>0.9973</td>
</tr>
</tbody>
</table>

9. All odd moments are equal to zero.

10. Skewness = 0 and Kurtosis = 3 in normal distribution.

**Illustration:**

Find the probability that the standard normal value lies between 0 and 1.5
As the mean, \( Z = 0 \).

To find the area between \( Z = 0 \) and \( Z = 1.5 \), look the area between 0 and 1.5, from the table. It is 0.4332 (shaded area)

**Illustration:**

The results of a particular examination are given below in a summary form:

<table>
<thead>
<tr>
<th>Result</th>
<th>Percentage of candidates</th>
</tr>
</thead>
<tbody>
<tr>
<td>Passed with distinction</td>
<td>10</td>
</tr>
<tr>
<td>Passed</td>
<td>60</td>
</tr>
<tr>
<td>Failed</td>
<td>30</td>
</tr>
</tbody>
</table>

It is known that a candidate gets plucked if he obtains less than 40 marks, out of 100 while he must obtain at least 75 marks in order to pass with distinction. Determine the mean and standard deviation of the distribution of marks assuming this to be normal.

**Solution:**

30% students get marks less than 40.

\[
40 - \frac{X}{\sigma} = -0.52 \quad \text{(from the table)}
\]

107
40 − X = −0.52σ  

10% students get more than 75

40% area = 75 − X = 1.28  

= 75 − X = 1.28σ

Subtract (ii) from (i)

40 − X = −0.52σ  

75 − X = 1.28σ  

-------------------------  
-35 = -1.8σ  
35 = 1.8σ  
1.80σ = 35

35  
σ = ------ = 19.4  
1.80

Mean

40 − X = −0.52 × (19.4)  

-X = -40 − 10.09 = 50.09
Illustration:

The scores observed by candidate in a certain test are normally distributed with mean 1000 and standard deviation 200. What per cent of candidates receive scores (i) less than 800, (ii) between 800 and 1200? (the area under the curve between $Z = 0$ and $Z = 1$ is 0.34134).

Solution: 

\[
Z = \frac{X - \mu}{\sigma}
\]

- $X = 1000; \sigma = 200$

i) For $X = 800$

\[
Z = \frac{800 - 1000}{200} = -1
\]

Area between $Z = -1$ and $Z = 0$ is 0.34134

Area for $Z = -1 = 0.5 - 0.34134 = 0.15866$

Therefore, the percentage = 0.15866 x 100 = 15.86%

ii) when, $X = 1200,$

\[
Z = \frac{1200 - 1000}{200} = 1
\]

Area between $Z = 0$ and $Z = 1$ is 0.34134

Area between $X = 400$ to $X = 600$

i.e.,

\[
Z = -1 \text{ and } Z = 1 \text{ is } 0.34134 + 0.34134 = 0.6826 = 68.26%
\]
3. TESTING OF HYPOTHESIS

3.1 Test of Significance for Large Samples

The test of significance for the large samples can be explained by the following assumptions:

i. The random sampling distribution of statistics is approximately normal.

ii. Sampling values are sufficiently close to the population value and can be used for the calculation of standard error of estimate.

1. The Standard Error Of Mean.

   In the case of large samples, when we are testing the significance of statistic, the concept of standard error is used. It measures only sampling errors. Sampling errors are involved in estimating a population parameter from a sample, instead of including all the essential information in the population.

   (i) when standard deviation of the population is known, the formula is

   \[
   \text{S.E. X} = \frac{\sigma_p}{\sqrt{n}}
   \]

   Where,

   \[
   \text{S.E.X} = \text{The standard error of the mean, } \sigma_p = \text{Standard deviation of the population, and } n = \text{ Number of observations in the sample.}
   \]

   (ii) when standard deviation of population is not known, we have to use the standard deviation of the sample in calculating standard error of mean.
The formula is

\[
\text{S.E. } X = \frac{\sigma \text{ (sample)}}{\sqrt{n}}
\]

Where,

\[
\sigma = \text{ Standard deviation of the sample, and } n = \text{ Sample size}
\]

**Illustration:**

A sample of 100 students from Pondicherry University was taken and their average was found to be 116 lbs with a standard deviation of 20 lbs. Could the mean weight of students in the population be 125 pounds?

**Solution:**

Let us take the hypothesis that there is no significant difference between the sample mean and the hypothetical population mean.

\[
\text{S.E. } X = \frac{\sigma}{\sqrt{n}} = \frac{20}{\sqrt{100}} = \frac{20}{10} = 2
\]

\[
\text{Difference} = \frac{125 - 116}{2} = \frac{9}{2} = 4.5
\]

Since, the difference is more than 2.58 S.E.(1% level) it could not have arisen due to fluctuations of sampling. Hence the mean weight of students in the population could not be 125 lbs.

### 3.2 Test Of Significance For Small Samples

If the sample size is less than 30, then those samples may be regarded as small samples. As a rule, the methods and the theory of large samples are not applicable to the small samples. The small samples are used in
testing a given hypothesis, to find out the observed values, which could have arisen by sampling fluctuations from some values given in advance. In a small sample, the investigator’s estimate will vary widely from sample to sample. An inference drawn from a smaller sample result is less precise than the inference drawn from a large sample result.

\[ t \text{-distribution will be employed, when the sample size is 30 or less and the population standard deviation is unknown.} \]

The formula is

\[
\frac{X - \mu}{\sigma} \cdot \sqrt{n}
\]

where,

\[
\sigma = \sqrt{\frac{\sum(X - \bar{X})^2}{n - 1}}
\]

**Illustration:**

The following results are obtained from a sample of 20 boxes of mangoes:

- Mean weight of contents = 490gms,
- Standard deviation of the weight = 9 gms.
- Could the sample come from a population having a mean of 500 gms?

**Solution:**

Let us take the hypothesis that \( \mu = 510 \) gms.

\[
\frac{X - \mu}{\sigma} \cdot \sqrt{n}
\]

\( X = 500; \mu = 510; \sigma = 10; n = 20. \)
\[ t = \frac{500 - 510}{\sqrt{20}} \]

\[ \text{Df} = 20 - 1 = 19 = (10/9) \sqrt{20} = (10/9) \times 4.47 = 4.47/9 = 4.96 \]

\[ \text{Df} = 19, t_{0.01} = 3.25 \]

The computed value is less than the table value. Hence, our null hypothesis is accepted.

4. **CHI-SQUARE TEST**

\( F, t \) and \( Z \) tests are based on the assumption that the samples were drawn from normally distributed populations. The testing procedure requires assumption about the type of population or parameters, and these tests are known as ‘parametric tests’.

There are many situations in which it is not possible to make any rigid assumption about the distribution of the population from which samples are being drawn. This limitation has led to the development of a group of alternative techniques known as non-parametric tests. Chi-square test of independence and goodness of fit is a prominent example of the use of non-parametric tests.

Though non-parametric theory developed as early as the middle of the nineteenth century, it was only after 1945 that non-parametric tests came to be used widely in sociological and psychological research. The main reasons for the increasing use of non-parametric tests in business research are:-

i. These statistical tests are distribution-free

ii. They are usually computationally easier to handle and understand than parametric tests; and

iii. They can be used with type of measurements that prohibit the use of parametric tests.

The \( \chi^2 \) test is one of the simplest and most widely used non-parametric tests in statistical work. It is defined as:
\[
\chi^2 = \frac{\sum(O - E)^2}{E}
\]

Where

\(O\) = the observed frequencies, and \(E\) = the expected frequencies.

Steps:

The steps required to determine the value of \(\chi^2\) are:

(i) Calculate the expected frequencies. In general the expected frequency for any cell can be calculated from the following equation:

\[
E = \frac{R \times C}{N}
\]

Where

\(E\) = Expected frequency, \(R\) = row’s total of the respective cell, \(C\) = column’s total of the respective cell and \(N\) = the total number of observations.

(ii) Take the difference between observed and expected frequencies and obtain the squares of these differences. Symbolically, it can be represented as \((O - E)^2\)

(iii) Divide the values of \((O - E)^2\) obtained in step (ii) by the respective expected frequency and obtain the total, which can be symbolically represented by \(\sum[(O - E)^2/E]\). This gives the value of \(\chi^2\) which can range from zero to infinity. If \(\chi^2\) is zero it means that the observed and expected frequencies completely coincide. The greater the discrepancy between the observed and expected frequencies, the greater shall be the value of \(\chi^2\).

The computed value of \(\chi^2\) is compared with the table value of \(\chi^2\) for given degrees of freedom at a certain specified level of significance. If at the stated level, the calculated value of \(\chi^2\) is less than the table value, the difference between theory and observation is not considered as significant.
The following observation may be made with regard to the $\chi^2$ distribution:-

i. The sum of the observed and expected frequencies is always zero. Symbolically, $\sum(O - E) = \sum O - \sum E = N - N = 0$

ii. The $\chi^2$ test depends only on the set of observed and expected frequencies and on degrees of freedom $v$. It is a non-parametric test.

iii. $\chi^2$ distribution is a limiting approximation of the multinomial distribution.

iv. Even though $\chi^2$ distribution is essentially a continuous distribution it can be applied to discrete random variables whose frequencies can be counted and tabulated with or without grouping.

**The Chi-Square Distribution**

For large sample sizes, the sampling distribution of $\chi^2$ can be closely approximated by a continuous curve known as the Chi-square distribution. The probability function of $\chi^2$ distribution is:

$$F(\chi^2) = C (\chi^2)^{(v/2 - 1)}e^{-\chi^2/2}$$

Where

- $e = 2.71828$, $v =$ number of degrees of freedom, $C =$ a constant depending only on $v$.

The $\chi^2$ distribution has only one parameter, $v$, the number of degrees of freedom. As in case of $t$-distribution there is a distribution for each different number of degrees of freedom. For very small number of degrees of freedom, the Chi-square distribution is severely skewed to the right. As the number of degrees of freedom increases, the curve rapidly becomes more symmetrical. For large values of $v$ the Chi-square distribution is closely approximated by the normal curve.
The following diagram gives $\chi^2$ distribution for 1, 5 and 10 degrees of freedom:

![Diagram of $\chi^2$ distribution](image)

It is clear from the given diagram that as the degrees of freedom increase, the curve becomes more and more symmetric. The Chi-square distribution is a probability distribution and the total area under the curve in each Chi-square distribution is unity.

**Properties of $\chi^2$ Distribution**

The main properties of $\chi^2$ distribution are:

(i) The mean of the $\chi^2$ distribution is equal to the number of degrees of freedom,

\[ X = \nu \]

i.e.,

\[ X = \nu \]

(ii) The variance of the $\chi^2$ distribution is twice the degrees of freedom, Variance $= 2\nu$

(iii) $\mu_1 = 0$,

(iv) $\mu_2 = 2\nu$,  

116
(v) \[ \mu_3 = 8v, \]

(vi) \[ \mu_4 = 48v + 12v^2. \]

\[ \begin{align*}
\frac{\mu_3^2}{8} &= \frac{64v^2}{8} \\
\beta_1 &= \frac{\mu_3^2}{8v^3} \frac{64v^2}{8} \\
\beta_3 &= \frac{\mu_3^2}{8v^3} \frac{64v^2}{8}
\end{align*} \]

(v) \[ \beta_3 \mu_3 = \frac{\mu_4}{4v^2} \frac{48v + 12v^2}{12} = 3 + \frac{\mu_4}{4v^2} \frac{48v + 12v^2}{12} \]

The table values of \( \chi^2 \) are available only up to 30 degrees of freedom. For degrees of freedom greater than 30, the distribution of \( \chi^2 \) approximates the normal distribution. For degrees of freedom greater than 30, the approximation is acceptable close. The mean of the distribution \( \sqrt{2} \chi^2 \) is \( \sqrt{2}v - 1 \), and the standard deviation is equal to 1. Thus the application of the test is simple, for deviation of \( \sqrt{2} \chi^2 \) from \( \sqrt{2}v - 1 \) may be interpreted as a normal deviate with units standard deviation. That is,

\[ Z = \sqrt{2} \chi^2 - \sqrt{2}v - 1 \]

Alternative Method Of Obtaining The Value of \( \chi^2 \)

In a 2x2 table where the cell frequencies and marginal totals are as below:

\[
\begin{array}{ccc}
  &  &  \\
  a & b & (a+b) \\
  c & d & (c+d) \\
  & (a+c) & (b+d) & N \\
\end{array}
\]

N is the total frequency and ad the larger cross-product, the value of \( \chi^2 \) can easily be obtained by the following formula:
\[ \chi^2 = \frac{N (ad - bc)^2}{(a + c) (b + d) (c + d) (a + b)} \]

or

\[ \chi^2 = \frac{N (ab - bc - \frac{1}{2}N)^2}{(a + c) (b + d) (c + d) (a + b)} \]

With Yate's corrections

**Conditions for Applying \( \chi^2 \) Test:**

The main conditions considered for employing the \( \chi^2 \) test are:

(i) \( N \) must be large enough to ensure the similarity between theoretically correct distribution and our sampling distribution of \( \chi^2 \).

(ii) No theoretical cell frequency should be small when the expected frequencies are too small. If it is so, then the value of \( \chi^2 \) will be overestimated and will result in too many rejections of the null hypothesis. To avoid making incorrect inferences, a general rule is followed that expected frequency of less than 5 in one cell of a contingency table is too small to use. When the table contains more than one cell with an expected frequency of less than 5 then add with the preceding or succeeding frequency so that the resulting sum is 5 or more. However, in doing so, we reduce the number of categories of data and will gain less information from contingency table.

(iii) The constraints on the cell frequencies if any should be linear, i.e., they should not involve square and higher powers of the frequencies such as \( \sum O = \sum E = N \).

**Uses of \( \chi^2 \) test:**

The main uses of \( \chi^2 \) test are:

i. **\( \chi^2 \) test as a test of independence.** With the help of \( \chi^2 \) test, we can find out whether two or more attributes are associated or not. Let’s assume that we have \( n \) observations classified according to some attributes.
We may ask whether the attributes are related or independent. Thus, we can find out whether there is any association between skin colour of husband and wife. To examine the attributes that are associated, we formulate the null hypothesis that there is no association against an alternative hypothesis and that there is an association between the attributes under study. If the calculated value of $\chi^2$ is less than the table value at a certain level of significance, we say that the result of the experiment provides no evidence for doubting the hypothesis. On the other hand, if the calculated value of $\chi^2$ is greater than the table value at a certain level of significance, the results of the experiment do not support the hypothesis.

ii. **$\chi^2$ test as a test of goodness of fit.** This is due to the fact that it enables us to ascertain how appropriately the theoretical distributions such as binomial, Poisson, Normal, etc., fit empirical distributions. When an ideal frequency curve whether normal or some other type is fitted to the data, we are interested in finding out how well this curve fits with the observed facts. A test of the concordance of the two can be made just by inspection, but such a test is obviously inadequate. Precision can be secured by applying the $\chi^2$ test.

iii. **$\chi^2$ test as a test of homogeneity.** The $\chi^2$ test of homogeneity is an extension of the chi-square test of independence. Tests of homogeneity are designed to determine whether two or more independent random samples are drawn from the same population or from different populations. Instead of one sample as we use with independence problem we shall now have 2 or more samples. For example, we may be interested in finding out whether or not university students of various levels, i.e., middle and richer poor income groups are homogeneous in performance in the examination.

**Illustration:**

In an anti-diabetes campaign in a certain area, a particular medicine, say x was administered to 812 persons out of a total population of 3248. The number of diabetes cases is shown below:
<table>
<thead>
<tr>
<th>Treatment</th>
<th>Diabetes</th>
<th>No Diabetes</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Medicine x</td>
<td>20</td>
<td>792</td>
<td>812</td>
</tr>
<tr>
<td>No Medicine x</td>
<td>220</td>
<td>2216</td>
<td>2436</td>
</tr>
<tr>
<td>Total</td>
<td>240</td>
<td>3008</td>
<td>3248</td>
</tr>
</tbody>
</table>

Discuss the usefulness of medicine x in checking malaria.

Solution:

Let us take the hypothesis that quinine is not effective in checking diabetes. Applying $\chi^2$ test:

$$\text{Expectation of (AB) = } \frac{(A) \times (B)}{N} = \frac{240 \times 812}{3248} = 60$$

Or $E_1$, i.e., expected frequency corresponding to first row and first column is 60. The bale of expected frequencies shall be:

<table>
<thead>
<tr>
<th></th>
<th>60</th>
<th>752</th>
<th>812</th>
</tr>
</thead>
<tbody>
<tr>
<td>180</td>
<td>180</td>
<td>2256</td>
<td>2436</td>
</tr>
<tr>
<td>240</td>
<td>3008</td>
<td>3248</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>O</th>
<th>E</th>
<th>(O – E)$^2$</th>
<th>(O – E)$^2$/E</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>60</td>
<td>1600</td>
<td>26.667</td>
</tr>
<tr>
<td>220</td>
<td>180</td>
<td>1600</td>
<td>8.889</td>
</tr>
<tr>
<td>792</td>
<td>752</td>
<td>1600</td>
<td>2.218</td>
</tr>
<tr>
<td>2216</td>
<td>2256</td>
<td>1600</td>
<td>0.709</td>
</tr>
</tbody>
</table>

$$[\Sigma(O – E)^2/E] = 38.593$$
\[ \chi^2 = \frac{\sum(O - E)^2}{E} = 38.593 \]

\[ V = (r - 1)(c - 1) = (2 - 1)(2 - 1) = 1 \]

For
\[ v = 1, \chi^2_{0.05} = 3.84 \]

The calculated value of \( \chi^2 \) is greater than the table value. The hypothesis is rejected. Hence medicine x is useful in checking malaria.

**Illustration:**

In an experiment on immunization of cattle from tuberculosis the following results were obtained:

<table>
<thead>
<tr>
<th></th>
<th>Affected</th>
<th>not affected</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inoculated</td>
<td>10</td>
<td>20</td>
</tr>
<tr>
<td>Not inoculated</td>
<td>15</td>
<td>5</td>
</tr>
</tbody>
</table>

Calculate \( \chi^2 \) and discuss the effect of vaccine in controlling susceptibility to tuberculosis (5% value of \( \chi^2 \) for one degree of freedom = 3.84).

**Solution:**

Let us take the hypothesis that the vaccine is not effective in controlling susceptibility to tuberculosis. Applying \( \chi^2 \) test:

\[ \chi^2 = \frac{(11 \times 5 - 20 \times 15)^2}{30 \times 20 \times 25 \times 25} = 8.3 \]

Since the calculated value of \( \chi^2 \) is greater than the table value the hypothesis is not true. We, therefore, conclude the vaccine is effective in controlling susceptibility to tuberculosis.

***
A BRIEF INTRODUCTION TO STATISTICAL APPLICATIONS

A manager in a business organization – whether in the top level, or the middle level, or the bottom level - has to perform an important role of decision making. For solving any organizational problem – which most of the times happens to be complex in nature -, he has to identify a set of alternatives, evaluate them and choose the best alternative. The experience, expertise, rationality and wisdom gained by the manager over a period of time will definitely stand in good stead in the evaluation of the alternatives available at his disposal. He has to consider several factors, sometimes singly and sometimes jointly, during the process of decision making. He has to deal with the data of not only his organization but also of other competing organizations.

It would be a challenging situation for a manager when he has to face so many variables operating simultaneously, something internal and something external. Among them, he has to identify the important variables or the dominating factors and he should be able to distinguish one factor from the other. He should be able to find which factors have similar characteristics and which factors stand apart. He should be able to know which factors have an inter play with each other and which factors remain independent. It would be advantageous to him to know whether there is any clear pattern followed by the variables under consideration. At times he may be required to have a good idea of the values that the variables would assume in future occasions. The task of a manager becomes all the more difficult in view of the risks and uncertainties surrounding the future events. It is imperative on the part of a manager to understand the impact of various policies and programmes on the development of the organization as well as the environment. Also he should be able to understand the impact of several of the environmental factors on his organization. Sometimes a manager has to take a single stage decision and at times he is called for to take a multistage decision on the basis of various factors operating in a situation.
Statistical analysis is a tool for a manager in the process of decision making by means of the data on hand. All managerial activities involve an analysis of data. Statistical approach would enable a manager to have a scientific guess of the future events also. Statistical methods are systematic and built by several experts on firmly established theories and consequently they would enable a manager to overcome the uncertainties associated with future occasions. However, statistical tools have their shortcomings too. The limitations do not reflect on the subject. Rather they shall be traced to the methods of data collection and recording of data. Even with highly sophisticated statistical methods, one may not arrive at valid conclusions if the data collected are devoid of representative character.

In any practical problem, one has to see whether the assumptions are reasonable or not, whether the data represents a wide spectrum, whether the data is adequate, whether all the conditions for the statistical tests have been fulfilled, etc. If one takes care of these aspects, it would be possible to arrive at better alternatives and more reliable solutions, thereby avoiding future shocks. While it is true that a statistical analysis, by itself, cannot solve all the problems faced by an organization, it will definitely enable a manager to comprehend the ground realities of the situation. It will for sure provide a foresight in the identification of the crucial variables and the key areas so that he can locate a set of possible solutions within his ambit. A manager has to have a proper blend of the statistical theories and practical wisdom and he shall always strive for a holistic approach to solve any organizational problem. A manager has to provide some safe-guarding measures against the limitations of the statistical tools. In the process he will be able to draw valid inferences thereby providing a clue as to the direction in which the organization shall move in future. He will be ably guided by the statistical results in the formulation of appropriate strategies for the organization. Further, he can prepare the organization to face the possible problems of business fluctuations in future and minimize the risks with the help of the early warning signals indicated by the relevant statistical tools.

A marketing manager of a company or a manager in a service organization will have occasions to come across the general public and consumers with several social and psychological variables which are difficult to be measured and quantified.
Depending on the situation and the requirement, a manager may have to deal with the data of just one variable (univariate data), or data on two variables (bivariate data) or data concerning several simultaneous variables (multivariate data).

The unit on hand addresses itself to the role of a manager as a decision maker with the help of data available with him. Different statistical techniques which are suitable for different requirements are presented in this unit in a simple style. A manager shall know the strengths and weaknesses of various statistical tools. He shall know which statistical tool would be the most appropriate in a particular context so that the organization will derive the maximum benefit out of it.

The interpretation of the results from statistical analysis occupies an important place. Statistics is concerned with the aggregates and not just the individual data items or isolated measurements of certain variables. Therefore the conclusions from a statistical study will be valid for a majority of the objects and normal situations only. There are always extreme cases in any problem and they have to be dealt with separately. Statistical tools will enable a manager to identify such outliers (abnormal cases or extreme variables) in a problem. A manager has to evaluate the statistical inferences, interpret them in the proper context and apply them in appropriate situations.

While in an actual research problem, one has to handle a large quantum of data, it is not possible to treat such voluminous data by a beginner in the subject. Keeping this point in mind, any numerical example in the present unit is based on a few data items only. It would be worthwhile to the budding managers to make a start in solving statistical problems by practicing the ones furnished in this unit.

The candidates are suggested to use hand calculators for solving statistical problems. There will be frequent occasions to use statistical tables of f-values furnished in this unit. The candidates are suggested to have with them a copy of the tables for easy, ready reference. The books and articles listed under the references may be consulted for further study or applications of statistical techniques in relevant research areas.
UNIT IV

1. Correlation And Regression Analysis

Lesson Outline

- The Concept Of Correlation
- Determination Of Simple Correlation Coefficient
- Properties Of Correlation Coefficient
- The Concept Of Rank Correlation
- Determination Of Rank Correlation Coefficient
- The Concept Of Regression
- The Principle Of Least Squares
- Normal Equations
- Determination Of Regression Equations

Learning Objectives

After reading this lesson you should be able to

- Understand the concept of correlation
- Calculate Simple correlation coefficient
- Understand the properties of correlation coefficient
- Understand the concept of rank correlation
- Calculate rank correlation coefficient
- Resolve ties in ranks
- Understand the concept of regression
- Determine regression equations
- Understand the managerial applications of correlation and regression
SIMPLE CORRELATION

Correlation

Correlation means the average relationship between two or more variables. When changes in the values of a variable affect the values of another variable, we say that there is a correlation between the two variables. The two variables may move in the same direction or in opposite directions. Simply because of the presence of correlation between two variables, we cannot jump to the conclusion that there is a cause-effect relationship between them. Sometimes, it may be due to chance also.

Simple correlation

We say that the correlation is simple if the comparison involves two variables only.

TYPES OF CORRELATION

Positive correlation

If two variables \( x \) and \( y \) move in the same direction, we say that there is a positive correlation between them. In this case, when the value of one variable increases, the value of the other variable also increases and when the value of one variable decreases, the value of the other variable also decreases. Eg. The age and height of a child.

Negative correlation

If two variables \( x \) and \( y \) move in opposite directions, we say that there is a negative correlation between them. i.e., when the value of one variable increases, the value of the other variable decreases and vice versa. Eg. The price and demand of a normal good.
The following diagrams illustrate positive and negative correlations between x and y.

**Positive Correlation**

If changes in two variables are in the same direction and the changes are in equal proportion, we say that there is a perfect positive correlation between them.

**Perfect Negative Correlation**

If changes in two variables are in opposite directions and the absolute values of changes are in equal proportion, we say that there is a perfect negative correlation between them.
Zero Correlation

If there is no relationship between the two variables, then the variables are said to be independent. In this case the correlation between the two variables is zero.

![Graph showing zero correlation](image)

Linear Correlation

If the quantum of change in one variable always bears a constant ratio to the quantum of change in the other variable, we say that the two variables have a linear correlation between them.

Coefficient of Correlation

The coefficient of correlation between two variables $X$, $Y$ is a measure of the degree of association (i.e., strength of relationship) between them. The coefficient of correlation is usually denoted by ‘$r$’.

Karl Pearson’s Coefficient Of Simple Correlation:

Let $N$ denote the number of pairs of observations of two variables $X$ and $Y$. The correlation coefficient $r$ between $X$ and $Y$ is defined by

$$ r = \frac{N \sum XY - (\sum X)(\sum Y)}{\sqrt{N \sum X^2 - (\sum X)^2} \sqrt{N \sum Y^2 - (\sum Y)^2}} $$
This formula is suitable for solving problems with hand calculators. To apply this formula, we have to calculate $\Sigma X, \Sigma Y, \Sigma XY, \Sigma X^2, \Sigma Y^2$.

**Properties Of Correlation Coefficient**

Let $r$ denote the correlation coefficient between two variables. $r \geq$ is interpreted using the following properties:

1. The value of $r$ ranges from $-1.0$ to $0.0$ or from $0.0$ to $1.0$
2. A value of $r = 1.0$ indicates that there exists perfect positive correlation between the two variables.
3. A value of $r = -1.0$ indicates that there exists perfect negative correlation between the two variables.
4. A value $r = 0.0$ indicates zero correlation i.e., it shows that there is no correlation at all between the two variables.
5. A positive value of $r$ shows a positive correlation between the two variables.
6. A negative value of $r$ shows a negative correlation between the two variables.
7. A value of $r = 0.9$ and above indicates a very high degree of positive correlation between the two variables.
8. A value of $-0.9 \geq r > -1.0$ shows a very high degree of negative correlation between the two variables.
9. For a reasonably high degree of positive correlation, we require $r$ to be from $0.75$ to $1.0$.
10. A value of $r$ from $0.6$ to $0.75$ may be taken as a moderate degree of positive correlation.

**Problem 1**

The following are data on Advertising Expenditure (in Rupees Thousand) and Sales (Rupees in lakhs) in a company.

Advertising Expenditure : 18 19 20 21 22 23  
Sales : 17 17 18 19 19 19

Determine the correlation coefficient between them and interpret the result.
Solution:

We have $N = 6$. Calculate $\Sigma X$, $\Sigma Y$, $\Sigma XY$, $\Sigma Y^2$, $\Sigma X^2$ as follows:

<table>
<thead>
<tr>
<th>$X$</th>
<th>$Y$</th>
<th>$XY$</th>
<th>$X^2$</th>
<th>$Y^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>18</td>
<td>17</td>
<td>306</td>
<td>324</td>
<td>289</td>
</tr>
<tr>
<td>19</td>
<td>17</td>
<td>323</td>
<td>361</td>
<td>289</td>
</tr>
<tr>
<td>20</td>
<td>18</td>
<td>360</td>
<td>400</td>
<td>324</td>
</tr>
<tr>
<td>21</td>
<td>19</td>
<td>399</td>
<td>441</td>
<td>361</td>
</tr>
<tr>
<td>22</td>
<td>19</td>
<td>418</td>
<td>484</td>
<td>361</td>
</tr>
<tr>
<td>23</td>
<td>19</td>
<td>437</td>
<td>529</td>
<td>361</td>
</tr>
<tr>
<td>Total:123</td>
<td>109</td>
<td>2243</td>
<td>2539</td>
<td>1985</td>
</tr>
</tbody>
</table>

The correlation coefficient $r$ between the two variables is calculated as follows:

$$r = \frac{N \sum XY - (\sum X)(\sum Y)}{\sqrt{N \sum X^2 - (\sum X)^2} \sqrt{N \sum Y^2 - (\sum Y)^2}}$$

$$r = \frac{6 \times 2243 - 123 \times 109}{\sqrt{6 \times 2539 - (123)^2} \sqrt{6 \times 1985 - (109)^2}}$$

$$= \frac{(13458 - 13407)}{\sqrt{(15234-15129)} \sqrt{(11910-11881)}}$$

$$= 51/\sqrt{105 \times 29} = 51/ (10.247 \times 5.365)$$

$$= 51/ 54.975$$

$$= 0.9277$$

Interpretation

The value of $r$ is 0.92. It shows that there is a high, positive correlation between the two variables ‘Advertising Expenditure’ and ‘Sales’. This provides a basis to consider some functional relationship between them.
Problem 2

Consider the following data on two variables X and Y.

<table>
<thead>
<tr>
<th>X</th>
<th>Y</th>
<th>XY</th>
<th>X²</th>
<th>Y²</th>
</tr>
</thead>
<tbody>
<tr>
<td>12</td>
<td>18</td>
<td>216</td>
<td>144</td>
<td>324</td>
</tr>
<tr>
<td>14</td>
<td>13</td>
<td>182</td>
<td>196</td>
<td>169</td>
</tr>
<tr>
<td>18</td>
<td>12</td>
<td>216</td>
<td>324</td>
<td>144</td>
</tr>
<tr>
<td>23</td>
<td>30</td>
<td>690</td>
<td>529</td>
<td>900</td>
</tr>
<tr>
<td>24</td>
<td>25</td>
<td>600</td>
<td>576</td>
<td>625</td>
</tr>
<tr>
<td>27</td>
<td>10</td>
<td>270</td>
<td>729</td>
<td>100</td>
</tr>
<tr>
<td>Total: 118</td>
<td>108</td>
<td>2174</td>
<td>2498</td>
<td>2262</td>
</tr>
</tbody>
</table>

The correlation coefficient between the two variables is  
\[ r = \frac{6 \times 2174 - (118 \times 108)}{\sqrt{6 \times 2498 - 118^2} \sqrt{6 \times 2262 - 108^2}} \]

\[ = \frac{13044 - 12744}{\sqrt{14988 - 13924} \sqrt{13572 - 11664}} \]

\[ = \frac{300}{\sqrt{1064} \sqrt{1908}} = \frac{300}{32.62 \times 43.68} \]

\[ = 0.2105 \]

Interpretation

The value of r is 0.21. Even though it is positive, the value of r is very less. Hence we conclude that there is no correlation between the two variables X and Y. Consequently we cannot construct any functional relational relationship between them.
Problem 3

Consider the following data on supply and price. Determine the correlation Coefficient between the two variables and interpret the result.

Supply : 11, 13, 17, 18, 22, 24, 26, 28
Price : 25, 32, 26, 25, 20, 17, 11, 10

Determine the correlation coefficient between the two variables and interpret the result.

Solution:

We have N = 8. Take X = Supply and Y = Price.

Calculate Σ X, ΣY, ΣXY, ΣX², ΣY² as follows:

<table>
<thead>
<tr>
<th>X</th>
<th>Y</th>
<th>XY</th>
<th>X²</th>
<th>Y²</th>
</tr>
</thead>
<tbody>
<tr>
<td>11</td>
<td>25</td>
<td>275</td>
<td>121</td>
<td>625</td>
</tr>
<tr>
<td>13</td>
<td>32</td>
<td>416</td>
<td>169</td>
<td>1024</td>
</tr>
<tr>
<td>17</td>
<td>26</td>
<td>442</td>
<td>289</td>
<td>676</td>
</tr>
<tr>
<td>18</td>
<td>25</td>
<td>450</td>
<td>324</td>
<td>625</td>
</tr>
<tr>
<td>22</td>
<td>20</td>
<td>440</td>
<td>484</td>
<td>400</td>
</tr>
<tr>
<td>24</td>
<td>17</td>
<td>408</td>
<td>576</td>
<td>289</td>
</tr>
<tr>
<td>26</td>
<td>11</td>
<td>286</td>
<td>676</td>
<td>121</td>
</tr>
<tr>
<td>28</td>
<td>10</td>
<td>280</td>
<td>784</td>
<td>100</td>
</tr>
<tr>
<td>Total:</td>
<td>159</td>
<td>166</td>
<td>2997</td>
<td>3423</td>
</tr>
</tbody>
</table>

The correlation coefficient between the two variables is

$r = \frac{8 \times 2997 - (159 \times 166)}{\sqrt{8 \times 3423 - 159^2} \sqrt{8 \times 3860 - 166^2}}$

$= \frac{23976 - 26394}{\sqrt{2103} \sqrt{3314}}$

$= \frac{-2418}{45.86 \times 57.57}$

$= -0.9159$
Interpretation

The value of r is -0.92. The negative sign in r shows that the two variables move in opposite directions. The absolute value of r is 0.92 which is very high. Therefore we conclude that there is high negative correlation between the two variables ‘Supply’ and ‘Price’.

Problem 4
Consider the following data on income and savings in Rs. Thousand.

Income : 50 51 52 55 56 58 60 62 65 66
Savings : 10 11 13 14 15 16 16 17 17

Determine the correlation coefficient between the two variables and interpret the result.

Solution:

We have N = 10. Take X = Income and Y = Savings.

Calculate $\Sigma X$, $\Sigma Y$, $\Sigma XY$, $\Sigma X^2$, $\Sigma Y^2$ as follows:

<table>
<thead>
<tr>
<th>X</th>
<th>Y</th>
<th>XY</th>
<th>X^2</th>
<th>Y^2</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>10</td>
<td>500</td>
<td>2500</td>
<td>100</td>
</tr>
<tr>
<td>51</td>
<td>11</td>
<td>561</td>
<td>2601</td>
<td>121</td>
</tr>
<tr>
<td>52</td>
<td>13</td>
<td>676</td>
<td>3025</td>
<td>196</td>
</tr>
<tr>
<td>55</td>
<td>14</td>
<td>770</td>
<td>3364</td>
<td>225</td>
</tr>
<tr>
<td>56</td>
<td>15</td>
<td>840</td>
<td>3600</td>
<td>256</td>
</tr>
<tr>
<td>58</td>
<td>15</td>
<td>870</td>
<td>3364</td>
<td>225</td>
</tr>
<tr>
<td>60</td>
<td>16</td>
<td>960</td>
<td>3600</td>
<td>256</td>
</tr>
<tr>
<td>62</td>
<td>16</td>
<td>992</td>
<td>3844</td>
<td>256</td>
</tr>
<tr>
<td>65</td>
<td>17</td>
<td>1105</td>
<td>4225</td>
<td>289</td>
</tr>
<tr>
<td>66</td>
<td>17</td>
<td>1122</td>
<td>4356</td>
<td>289</td>
</tr>
<tr>
<td>Total: 575</td>
<td>144</td>
<td>8396</td>
<td>33355</td>
<td>2126</td>
</tr>
</tbody>
</table>

The correlation coefficient between the two variables is $r =$

$$\frac{10 \times 8396 - (575 \times 144)}{\sqrt{(10 \times 33355 - 575^2) \sqrt{(10 \times 2126 - 144^2)}}}$$

$$= \frac{83960 - 82800}{\sqrt{2925 \times 524}}$$

$$= 1160 / (54.08 \times 22.89)$$

$$= 1160 / 1237.89 = 0.9371$$
Interpretation

The value of \( r \) is 0.93. The positive sign in \( r \) shows that the two variables move in the same direction. The value of \( r \) is very high. Therefore we conclude that there is high positive correlation between the two variables ‘Income’ and ‘Savings’. As a result, we can construct a functional relationship between them.

RANK CORRELATION

Spearman's rank correlation coefficient

If ranks can be assigned to pairs of observations for two variables \( X \) and \( Y \), then the correlation between the ranks is called the rank correlation coefficient. It is usually denoted by the symbol \( \rho \) (rho). It is given by the formula

\[
\rho = 1 - \frac{6 \sum D^2}{N^3 - N}
\]

where

\[
D = \text{difference between the corresponding ranks of } X \text{ and } Y
\]

\[
= R_x - R_y
\]

and \( N \) is the total number of pairs of observations of \( X \) and \( Y \).
Alpha Recruiting Agency shortlisted 10 candidates for final selection. They were examined in written and oral communication skills. They were ranked as follows:

<table>
<thead>
<tr>
<th>Candidate’s Serial no.</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rank in written communication</td>
<td>8</td>
<td>7</td>
<td>2</td>
<td>10</td>
<td>3</td>
<td>5</td>
<td>1</td>
<td>9</td>
<td>6</td>
<td>4</td>
</tr>
<tr>
<td>Rank in oral communication</td>
<td>10</td>
<td>7</td>
<td>2</td>
<td>6</td>
<td>5</td>
<td>4</td>
<td>1</td>
<td>9</td>
<td>8</td>
<td>3</td>
</tr>
</tbody>
</table>

Find out whether there is any correlation between the written and oral communication skills of the shortlisted candidates.

**Solution:**

Take $X = $ Written Communication Skill and $Y = $ Oral Communication Skill.

<table>
<thead>
<tr>
<th>RANK OF $X$: $R_1$</th>
<th>RANK OF $Y$: $R_2$</th>
<th>$D = R_1 - R_2$</th>
<th>$D^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>10</td>
<td>-2</td>
<td>4</td>
</tr>
<tr>
<td>7</td>
<td>7</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>10</td>
<td>6</td>
<td>4</td>
<td>16</td>
</tr>
<tr>
<td>3</td>
<td>5</td>
<td>-2</td>
<td>4</td>
</tr>
<tr>
<td>5</td>
<td>4</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>9</td>
<td>9</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>6</td>
<td>8</td>
<td>-2</td>
<td>4</td>
</tr>
<tr>
<td>4</td>
<td>3</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

Total: 30

We have $N = 10$. The rank correlation coefficient is

$$
p = 1 - \frac{6 \sum D^2}{N^3 - N}
$$

$$
= 1 - \frac{6 \times 30}{(1000 - 10)}
$$
\[ r = 1 - \frac{180}{990} \]
\[ = 1 - 0.18 \]
\[ = 0.82 \]

**Inference:**

From the value of \( r \), it is inferred that there is a high, positive rank correlation between the written and oral communication skills of the short listed candidates.

**Problem 6**

The following are the ranks obtained by 10 workers in abc company on the basis of their length of service and efficiency.

<table>
<thead>
<tr>
<th>Ranking as per service</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rank as per efficiency</td>
<td>2</td>
<td>3</td>
<td>6</td>
<td>5</td>
<td>1</td>
<td>10</td>
<td>7</td>
<td>9</td>
<td>8</td>
<td>4</td>
</tr>
</tbody>
</table>

Find out whether there is any correlation between the ranks obtained by the workers as per the two criteria.

**Solution:**

Take \( X = \) Length of Service and \( Y = \) Efficiency.

<table>
<thead>
<tr>
<th>Rank of ( X: R_1 )</th>
<th>RANK OF ( Y: R_2 )</th>
<th>( D = R_1 - R_2 )</th>
<th>( D^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>-1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>-1</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>6</td>
<td>-3</td>
<td>9</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
<td>-1</td>
<td>1</td>
</tr>
<tr>
<td>5</td>
<td>1</td>
<td>4</td>
<td>16</td>
</tr>
<tr>
<td>6</td>
<td>10</td>
<td>-4</td>
<td>16</td>
</tr>
<tr>
<td>7</td>
<td>7</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>8</td>
<td>9</td>
<td>-1</td>
<td>1</td>
</tr>
<tr>
<td>9</td>
<td>8</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>10</td>
<td>4</td>
<td>6</td>
<td>36</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td><strong>Total</strong></td>
</tr>
</tbody>
</table>
We have \( N = 10 \). The rank correlation coefficient is
\[
\rho = 1 - \left\{ 6 \sum D^2 / (N^3 - N) \right\} \\
= 1 - \left\{ 6 \times 82 / (1000 - 10) \right\} \\
= 1 - \left( \frac{492}{990} \right) \\
= 1 - 0.497 \\
= 0.503
\]

**Inference:**
The rank correlation coefficient is not high.

**Problem 7 (conversion of scores into ranks)**

Calculate the rank correlation to determine the relationship between equity shares and preference shares given by the following data on their price.

<table>
<thead>
<tr>
<th>Equity share</th>
<th>90.0</th>
<th>92.4</th>
<th>98.5</th>
<th>98.3</th>
<th>95.4</th>
<th>91.3</th>
<th>98.0</th>
<th>92.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Preference share</td>
<td>76.0</td>
<td>74.2</td>
<td>75.0</td>
<td>77.4</td>
<td>78.3</td>
<td>78.8</td>
<td>73.2</td>
<td>76.5</td>
</tr>
</tbody>
</table>

**Solution:**

From the given data on share price, we have to find out the ranks for equity shares and preference shares.

**Step 1.**
First, consider the equity shares and arrange them in descending order of their price as 1,2,…,8. We have the following ranks:

<table>
<thead>
<tr>
<th>Equity share</th>
<th>98.5</th>
<th>98.3</th>
<th>98.0</th>
<th>95.4</th>
<th>92.4</th>
<th>92.0</th>
<th>91.3</th>
<th>90.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rank</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
<td>8</td>
</tr>
</tbody>
</table>

**Step 2.**
Next, take the preference shares and arrange them in descending order of their price as 1,2,…,8. We obtain the following ranks:
Step 3.
Calculation of $D^2$:

Fit the given data with the correct rank. Take $X = \text{Equity share}$ and $Y = \text{Preference share}$. We have the following table:

<table>
<thead>
<tr>
<th>Preference share</th>
<th>78.8</th>
<th>78.3</th>
<th>77.4</th>
<th>76.5</th>
<th>76.0</th>
<th>75.0</th>
<th>74.2</th>
<th>73.2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rank</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
<td>8</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>X</th>
<th>Y</th>
<th>Rank of X: $R_1$</th>
<th>Rank of Y: $R_2$</th>
<th>$D = R_1 - R_2$</th>
<th>$D^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>90.0</td>
<td>76.0</td>
<td>8</td>
<td>5</td>
<td>3</td>
<td>9</td>
</tr>
<tr>
<td>92.4</td>
<td>74.2</td>
<td>5</td>
<td>7</td>
<td>-2</td>
<td>4</td>
</tr>
<tr>
<td>98.5</td>
<td>75.0</td>
<td>1</td>
<td>6</td>
<td>-5</td>
<td>25</td>
</tr>
<tr>
<td>98.3</td>
<td>77.4</td>
<td>2</td>
<td>3</td>
<td>-1</td>
<td>1</td>
</tr>
<tr>
<td>95.4</td>
<td>78.3</td>
<td>4</td>
<td>2</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>91.3</td>
<td>78.8</td>
<td>7</td>
<td>1</td>
<td>6</td>
<td>36</td>
</tr>
<tr>
<td>98.0</td>
<td>73.2</td>
<td>3</td>
<td>8</td>
<td>-5</td>
<td>25</td>
</tr>
<tr>
<td>92.0</td>
<td>76.5</td>
<td>6</td>
<td>4</td>
<td>2</td>
<td>4</td>
</tr>
</tbody>
</table>

Total 108

Step 4.
Calculation of $\rho$:

We have $N = 8$. The rank correlation coefficient is

$$\rho = 1 - \frac{6 \sum D^2}{(N^3 - N)}$$

$$= 1 - \left\{ \frac{6 \times 108}{(512 - 8)} \right\}$$

$$= 1 - \left( \frac{648}{504} \right)$$

$$= 1 - 1.29$$

$$= -0.29$$
**Inference:**

From the value of $\rho$, it is inferred that the equity shares and preference shares under consideration are negatively correlated. However, the absolute value of $\rho$ is 0.29 which is not even moderate.

**Problem 8**

Three managers evaluate the performance of 10 sales persons in an organization and award ranks to them as follows:

<table>
<thead>
<tr>
<th>Sales Person</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rank Awarded by Manager I</td>
<td>8</td>
<td>7</td>
<td>6</td>
<td>1</td>
<td>5</td>
<td>9</td>
<td>10</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>Rank Awarded by Manager II</td>
<td>7</td>
<td>8</td>
<td>4</td>
<td>6</td>
<td>5</td>
<td>10</td>
<td>9</td>
<td>3</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>Rank Awarded by Manager III</td>
<td>4</td>
<td>5</td>
<td>1</td>
<td>8</td>
<td>9</td>
<td>10</td>
<td>6</td>
<td>7</td>
<td>3</td>
<td>2</td>
</tr>
</tbody>
</table>

Determine which two managers have the nearest approach in the evaluation of the performance of the sales persons.

**Solution:**

<table>
<thead>
<tr>
<th>Sales Person</th>
<th>Manager I Rank: $R_1$</th>
<th>Manager II Rank: $R_2$</th>
<th>Manager III Rank: $R_3$</th>
<th>$(R_1-R_2)^2$</th>
<th>$(R_1-R_3)^2$</th>
<th>$(R_2-R_3)^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>8</td>
<td>7</td>
<td>4</td>
<td>1</td>
<td>16</td>
<td>9</td>
</tr>
<tr>
<td>2</td>
<td>7</td>
<td>8</td>
<td>5</td>
<td>1</td>
<td>4</td>
<td>9</td>
</tr>
<tr>
<td>3</td>
<td>6</td>
<td>4</td>
<td>1</td>
<td>4</td>
<td>25</td>
<td>9</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>6</td>
<td>8</td>
<td>25</td>
<td>49</td>
<td>4</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
<td>5</td>
<td>9</td>
<td>0</td>
<td>16</td>
<td>16</td>
</tr>
<tr>
<td>6</td>
<td>9</td>
<td>10</td>
<td>10</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>7</td>
<td>10</td>
<td>9</td>
<td>6</td>
<td>1</td>
<td>16</td>
<td>9</td>
</tr>
</tbody>
</table>
We have $N = 10$. The rank correlation coefficient between managers I and II is

$$\rho = 1 - \{ 6 \sum D^2 / (N^3 - N) \}$$

$$= 1 - \{ 6 \times 44 / (1000 - 10) \}$$

$$= 1 - (264 / 990)$$

$$= 1 - 0.27$$

$$= 0.73$$

The rank correlation coefficient between managers I and III is

$$1 - \{ 6 \times 156 / (1000 - 10) \}$$

$$= 1 - (936 / 990)$$

$$= 1 - 0.95$$

$$= 0.05$$

The rank correlation coefficient between managers II and III is

$$1 - \{ 6 \times 74 / (1000 - 10) \}$$

$$= 1 - (444 / 990)$$

$$= 1 - 0.44$$

$$= 0.56$$

**Inference:**

Comparing the 3 values of $\rho$, it is inferred that Managers I and II have the nearest approach in the evaluation of the performance of the sales persons.

**Repeated values: Resolving ties in ranks**

When ranks are awarded to candidates, it is possible that certain candidates obtain equal ranks. For example, if two or three, or four
candidates secure equal ranks, a procedure that can be followed to resolve the ties is described below.

We follow the **Average Rank Method**. If there are \( n \) items, arrange them in ascending order or descending order and give ranks 1, 2, 3, ..., \( n \). Then look at those items which have equal values. For such items, take the average ranks.

If there are two items with equal values, their ranks will be two consecutive integers, say \( s \) and \( s + 1 \). Their average is \( \frac{s + (s+1)}{2} \). Assign this rank to both items. Note that we allow ranks to be fractions also.

If there are three items with equal values, their ranks will be three consecutive integers, say \( s \), \( s + 1 \) and \( s + 2 \). Their average is \( \frac{s + (s+1) + (s+2)}{3} = \frac{3s + 3}{3} = s + 1 \). Assign this rank to all the three items. A similar procedure is followed if four or more number of items has equal values.

**Correction term for \( \rho \) when ranks are tied**

Consider the formula for rank correlation coefficient. We have

\[
\rho = 1 - \frac{6 \sum D^2}{N^3 - N}
\]

If there is a tie involving \( m \) items, we have to add \( \frac{m^3 - m}{12} \) to the term \( D^2 \) in \( \rho \). We have to add as many terms like \( (m^3 - m) / 12 \) as there are ties.

Let us calculate the correction terms for certain values of \( m \). These are provided in the following table.
Illustrative examples:

If there is a tie involving 2 items, then the correction term is 0.5

If there are 2 ties involving 2 items each, then the correction term is $0.5 + 0.5 = 1$

If there are 3 ties with 2 items each, then the correction term is $0.5 + 0.5 + 0.5 = 1.5$

If there is a tie involving 3 items, then the correction term is 2

If there are 2 ties involving 3 items each, then the correction term is $2 + 2 = 4$

If there is a tie with 2 items and another tie with 3 items, then the correction term is $0.5 + 2 = 2.5$

If there are 2 ties with 2 items each and another tie with 3 items, then the correction term is $0.5 + 0.5 + 2 = 3$

<table>
<thead>
<tr>
<th>$m$</th>
<th>$m^3$</th>
<th>$m^3 - m$</th>
<th>Correction term</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>8</td>
<td>6</td>
<td>0.5</td>
</tr>
<tr>
<td>3</td>
<td>27</td>
<td>24</td>
<td>2</td>
</tr>
<tr>
<td>4</td>
<td>64</td>
<td>60</td>
<td>5</td>
</tr>
<tr>
<td>5</td>
<td>125</td>
<td>120</td>
<td>10</td>
</tr>
</tbody>
</table>
Problem 9: Resolving ties in ranks

The following are the details of ratings scored by two popular insurance schemes. Determine the rank correlation coefficient between them.

<table>
<thead>
<tr>
<th>Scheme I</th>
<th>80</th>
<th>80</th>
<th>83</th>
<th>84</th>
<th>87</th>
<th>87</th>
<th>89</th>
<th>90</th>
</tr>
</thead>
<tbody>
<tr>
<td>Scheme II</td>
<td>55</td>
<td>56</td>
<td>57</td>
<td>57</td>
<td>57</td>
<td>58</td>
<td>59</td>
<td>60</td>
</tr>
</tbody>
</table>

Solution:

From the given values, we have to determine the ranks.

Step 1.

Arrange the scores for Insurance Scheme I in descending order and rank them as 1,2,3,…,8.

<table>
<thead>
<tr>
<th>Scheme I Score</th>
<th>90</th>
<th>89</th>
<th>87</th>
<th>87</th>
<th>84</th>
<th>83</th>
<th>80</th>
<th>80</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rank</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
<td>8</td>
</tr>
</tbody>
</table>

The score 87 appears twice. The corresponding ranks are 3, 4. Their average is \( (3 + 4) / 2 = 3.5 \). Assign this rank to the two equal scores in Scheme I.

The score 80 appears twice. The corresponding ranks are 7, 8. Their average is \( (7 + 8) / 2 = 7.5 \). Assign this rank to the two equal scores in Scheme I.

The revised ranks for Insurance Scheme I are as follows:

<table>
<thead>
<tr>
<th>Scheme I Score</th>
<th>90</th>
<th>89</th>
<th>87</th>
<th>87</th>
<th>84</th>
<th>83</th>
<th>80</th>
<th>80</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rank</td>
<td>1</td>
<td>2</td>
<td>3.5</td>
<td>3.5</td>
<td>5</td>
<td>6</td>
<td>7.5</td>
<td>7.5</td>
</tr>
</tbody>
</table>
Step 2.

Arrange the scores for Insurance Scheme II in descending order and rank them as 1, 2, 3, ..., 8.

<table>
<thead>
<tr>
<th>Scheme II Score</th>
<th>60</th>
<th>59</th>
<th>58</th>
<th>57</th>
<th>57</th>
<th>57</th>
<th>56</th>
<th>55</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rank</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
<td>8</td>
</tr>
</tbody>
</table>

The score 57 appears thrice. The corresponding ranks are 4, 5, 6. Their average is \((4 + 5 + 6) / 3 = 15 / 3 = 5\). Assign this rank to the three equal scores in Scheme II.

The revised ranks for Insurance Scheme II are as follows:

<table>
<thead>
<tr>
<th>Scheme II Score</th>
<th>60</th>
<th>59</th>
<th>58</th>
<th>57</th>
<th>57</th>
<th>57</th>
<th>56</th>
<th>55</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rank</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>5</td>
<td>5</td>
<td>5</td>
<td>7</td>
<td>8</td>
</tr>
</tbody>
</table>

Step 3.

**Calculation of** \(D^2\): Assign the revised ranks to the given pairs of values and calculate \(D^2\) as follows:

<table>
<thead>
<tr>
<th>Scheme I Score</th>
<th>Scheme II Score</th>
<th>Scheme I Rank: (R_1)</th>
<th>Scheme II Rank: (R_2)</th>
<th>(D=R_1- R_2)</th>
<th>(D^2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>80</td>
<td>55</td>
<td>7.5</td>
<td>8</td>
<td>-0.5</td>
<td>0.25</td>
</tr>
<tr>
<td>80</td>
<td>56</td>
<td>7.5</td>
<td>7</td>
<td>0.5</td>
<td>0.25</td>
</tr>
<tr>
<td>83</td>
<td>57</td>
<td>6</td>
<td>5</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>84</td>
<td>57</td>
<td>5</td>
<td>5</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>87</td>
<td>57</td>
<td>3.5</td>
<td>5</td>
<td>-1.5</td>
<td>2.25</td>
</tr>
<tr>
<td>87</td>
<td>58</td>
<td>3.5</td>
<td>3</td>
<td>0.5</td>
<td>0.25</td>
</tr>
<tr>
<td>89</td>
<td>59</td>
<td>2</td>
<td>2</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>90</td>
<td>60</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

**Total** 4
Step 4.

Calculation of $\rho$:

We have $N = 8$.

Since there are 2 ties with 2 items each and another tie with 3 items, the correction term is $0.5 + 0.5 + 2$.

The rank correlation coefficient is

$$\rho = 1 - \left\{ \frac{6 \sum D^2 + (1/2) + (1/2) + 2}{(N^3 - N)} \right\}$$

$$= 1 - \{ 6 (4.05+0.5+0.5+2) / (512 - 8) \} = 1 - (6 x 7 / 504) = 1 - (42/504)$$

$$= 1 - 0.083 = 0.917$$

Inference:

It is inferred that the two insurance schemes are highly, positively correlated.

**REGRESSION**

In the pairs of observations, if there is a cause and effect relationship between the variables $X$ and $Y$, then the average relationship between these two variables is called regression, which means “stepping back” or “return to the average”. The linear relationship giving the best mean value of a variable corresponding to the other variable is called a **regression line or line of the best fit**. The regression of $X$ on $Y$ is different from the regression of $Y$ on $X$. Thus, there are two equations of regression and the two regression lines are given as follows:

Regression of $Y$ on $X$: $Y - \bar{Y} = b_{yx} (X - \bar{X})$

Regression of $X$ on $Y$: $X - \bar{X} = b_{xy} (Y - \bar{Y})$

Where $\bar{X}$, $\bar{Y}$ are the means of $X$, $Y$ respectively.

**Result:**

Let $\sigma_x$, $\sigma_y$ denote the standard deviations of $x$, $y$ respectively. We have the following result.
\[ b_{y|x} = r \frac{\sigma_y}{\sigma_x} \quad \text{and} \quad b_{x|y} = r \frac{\sigma_x}{\sigma_y} \]
\[ \therefore r^2 = b_{y|x} b_{x|y} \quad \text{and so} \quad r = \sqrt{b_{y|x} b_{x|y}} \]

**Result:**

The coefficient of correlation \( r \) between \( X \) and \( Y \) is the square root of the product of the \( b \) values in the two regression equations. We can find \( r \) by this way also.

**Application**

The method of regression is very much useful for business forecasting.

**PRINCIPLE OF LEAST SQUARES**

Let \( x, y \) be two variables under consideration. Out of them, let \( x \) be an independent variable and let \( y \) be a dependent variable, depending on \( x \). We desire to build a functional relationship between them. For this purpose, the first and foremost requirement is that \( x, y \) have a high degree of correlation. If the correlation coefficient between \( x \) and \( y \) is moderate or less, we shall not go ahead with the task of fitting a functional relationship between them.

Suppose there is a high degree of correlation (positive or negative) between \( x \) and \( y \). Suppose it is required to build a linear relationship between them i.e., we want a regression of \( y \) on \( x \).

Geometrically speaking, if we plot the corresponding values of \( x \) and \( y \) in a 2-dimensional plane and join such points, we shall obtain a straight line. However, hardly we can expect all the pairs \((x, y)\) to lie on a straight line. We can consider several straight lines which are, to some extent, near all the points \((x, y)\). Consider one line. An observation \((x, y_i)\) may be either above the line of consideration or below the line. Project this point on the \( x \)-axis. It will meet the straight line at the point \((x_i, y_i,e)\). Here the theoretical value (or the expected value) of the variable is \( y_i,e \) while the
observed value is $y_1$. When there is a difference between the expected and observed values, there appears an error. This error is $E_1 = y_1 - y_1$. This is positive if $(x_1, y_1)$ is a point above the line and negative if $(x_1, y_1)$ is a point below the line. For the $n$ pairs of observations, we have the following $n$ quantities of error:

$$E_1 = y_1 - y_1,$$
$$E_2 = y_2 - y_2,$$
$$E_n = y_n - y_n.$$

Some of these quantities are positive while the remaining ones are negative. However, the squares of all these quantities are positive.

i.e.,

$$E_1^2 = (y_1 - y_1)^2 \geq 0, E_2^2 = (y_2 - y_2)^2 \geq 0, \ldots, E_n^2 = (y_n - y_n)^2 \geq 0.$$

Hence the sum of squares of errors (SSE) = $E_1^2 + E_2^2 + \ldots + E_n^2$

$$= (y_1 - y_1)^2 + (y_2 - y_2)^2 + \ldots + (y_n - y_n)^2 \geq 0.$$
Among all those straight lines which are somewhat near to the
given observations
\((x_1, y_1), (x_2, y_2), \ldots, (x_n, y_n)\), we consider that straight line as the ideal one
for which the sse is the least. Since the ideal straight line giving regression
of \(y\) on \(x\) is based on this concept, we call this principle as the **Principle of
least squares.**

**Normal equations**

Suppose we have to fit a straight line to the \(n\) pairs of observations
\((x_1, y_1), (x_2, y_2), \ldots, (x_n, y_n)\). Suppose the equation of straight line finally
comes as
\[
Y = a + b X                      \quad (1)
\]
Where
\(a, b\) are constants to be determined. Mathematically speaking, when
we require finding the equation of a straight line, two distinct points on
the straight line are sufficient. However, a different approach is followed
here. We want to include all the observations in our attempt to build a
straight line. Then all the \(n\) observed points \((x, y)\) are required to satisfy
the relation

\((1)\). Consider the summation of all such terms. We get
\[
\Sigma y = \Sigma (a + b x) = \Sigma (a .1 + b x) = (\Sigma a.1) + (\Sigma b x) = a (\Sigma 1) + b (\Sigma x).
\]
i.e.
\[
\Sigma y = an + b (\Sigma x) \quad (2)
\]

To find two quantities \(a\) and \(b\), we require two equations. We have
obtained one equation i.e., \((2)\). We need one more equation. For this
purpose, multiply both sides of \((1)\) by
\[
x. \quad \text{We obtain} \quad x y = ax + bx^2.
\]
Consider the summation of all such terms. We get
\[
\Sigma x y = \Sigma (ax + bx^2) = (\Sigma a x) + (\Sigma bx^2)
\]
i.e.,

\[ \sum x y = a (\sum x) + b (\sum x^2) \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \}
Problem 10

Consider the following data on sales and profit.

<table>
<thead>
<tr>
<th>X</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
</tr>
</thead>
<tbody>
<tr>
<td>Y</td>
<td>2</td>
<td>4</td>
<td>5</td>
<td>5</td>
<td>3</td>
<td>8</td>
<td>7</td>
</tr>
</tbody>
</table>

Determine the regression of profit on sales.

Solution:

We have N = 7. Take X = Sales, Y = Profit.

Calculate $\Sigma X$, $\Sigma Y$, $\Sigma XY$, $\Sigma X^2$ as follows:

<table>
<thead>
<tr>
<th>X</th>
<th>Y</th>
<th>XY</th>
<th>$X^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>2</td>
<td>10</td>
<td>25</td>
</tr>
<tr>
<td>6</td>
<td>4</td>
<td>24</td>
<td>36</td>
</tr>
<tr>
<td>7</td>
<td>5</td>
<td>35</td>
<td>49</td>
</tr>
<tr>
<td>8</td>
<td>5</td>
<td>40</td>
<td>64</td>
</tr>
<tr>
<td>9</td>
<td>3</td>
<td>27</td>
<td>81</td>
</tr>
<tr>
<td>10</td>
<td>8</td>
<td>80</td>
<td>100</td>
</tr>
<tr>
<td>11</td>
<td>7</td>
<td>77</td>
<td>121</td>
</tr>
<tr>
<td>Total: 56</td>
<td>34</td>
<td>293</td>
<td>476</td>
</tr>
</tbody>
</table>

$$a = \left\{ \left( \Sigma x^2 \right) \left( \Sigma y \right) - \left( \Sigma x \right) \left( \Sigma x \; y \right) \right\} / n \left( \Sigma x^2 \right) - \left( \Sigma x \right)^2$$

$$= (476 \times 34 - 56 \times 293) / (7 \times 476 - 56^2)$$

$$= (16184 - 16408) / (3332 - 3136)$$

$$= -224 / 196$$

$$= -1.1429$$
\[
b = \frac{n (\sum xy) - (\sum x)(\sum y)}{n (\sum x^2) - (\sum x)^2}
\]
\[
= \frac{(7 \times 293 - 56 \times 34)}{196} = \frac{(2051 - 1904)}{196}
\]
\[
= 147 / 196
\]
\[
= 0.75
\]

The regression of Y on X is given by the equation

\[
Y = a + b X
\]

I.e.,

\[
Y = -1.14 + 0.75 X
\]

**Problem 11**

The following are the details of income and expenditure of 10 households.

<table>
<thead>
<tr>
<th>Income</th>
<th>40</th>
<th>70</th>
<th>50</th>
<th>60</th>
<th>80</th>
<th>50</th>
<th>90</th>
<th>40</th>
<th>60</th>
<th>60</th>
</tr>
</thead>
<tbody>
<tr>
<td>Expenditure</td>
<td>25</td>
<td>60</td>
<td>45</td>
<td>50</td>
<td>45</td>
<td>20</td>
<td>55</td>
<td>30</td>
<td>35</td>
<td>30</td>
</tr>
</tbody>
</table>

Determine the regression of expenditure on income and estimate the expenditure when the income is 65.

**Solution:**

We have N = 10. Take X = Income, Y = Expenditure

Calculate \( \sum X, \sum y, \sum xy, \sum x^2 \) as follows:

<table>
<thead>
<tr>
<th>X</th>
<th>Y</th>
<th>XY</th>
<th>X²</th>
</tr>
</thead>
<tbody>
<tr>
<td>40</td>
<td>25</td>
<td>1000</td>
<td>1600</td>
</tr>
<tr>
<td>70</td>
<td>60</td>
<td>4200</td>
<td>4900</td>
</tr>
<tr>
<td>50</td>
<td>45</td>
<td>2250</td>
<td>2500</td>
</tr>
</tbody>
</table>
\[
a = \frac{(\sum x^2)(\sum y) - (\sum x)(\sum xy)}{n(\sum x^2) - (\sum x)^2}
\]

\[
a = \frac{(38400 \times 395 - 600 \times 25100)}{10 \times 38400 - 600^2}
\]

\[
a = \frac{15168000 - 15060000}{384000 - 360000}
\]

\[
a = 108000 / 24000
\]

\[
a = 4.5
\]

\[
b = \frac{n(\sum xy) - (\sum x)(\sum y)}{n(\sum x^2) - (\sum x)^2}
\]

\[
b = \frac{(10 \times 25100 - 600 \times 395)}{24000}
\]

\[
b = \frac{251000 - 237000}{24000}
\]

\[
b = 14000 / 24000
\]

\[
b = 0.58
\]

The regression of \( y \) on \( x \) is given by the equation

\[
Y = a + b X
\]

i.e.,

\[
Y = 4.5 + 0.583 \times X
\]
To estimate the expenditure when income is 65:

Take $X = 65$ in the above equation. Then we get

$$Y = 4.5 + 0.583 \times 65$$

$$= 4.5 + 37.895$$

$$= 42.395$$

$$= 42 \text{ (approximately).}$$

Problem 12

Consider the following data on occupancy rate and profit of a hotel.

<table>
<thead>
<tr>
<th>Occupancy rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>40</td>
</tr>
<tr>
<td>Profit</td>
</tr>
<tr>
<td>50</td>
</tr>
</tbody>
</table>

Determine the regressions of

(i) profit on occupancy rate and
(ii) occupancy rate on profit.

Solution:

We have $N = 10$. Take $X = \text{Occupancy Rate}$, $Y = \text{Profit}$.

Note that in Problems 10 and 11, we wanted only one regression line and so we did not take $\sum Y^2$. Now we require two regression lines. Therefore,
Calculate $\sum X$, $\sum Y$, $\sum XY$, $\sum X^2$, $\sum Y^2$.

<table>
<thead>
<tr>
<th>X</th>
<th>Y</th>
<th>XY</th>
<th>X2</th>
<th>Y2</th>
</tr>
</thead>
<tbody>
<tr>
<td>40</td>
<td>50</td>
<td>2000</td>
<td>1600</td>
<td>2500</td>
</tr>
<tr>
<td>45</td>
<td>55</td>
<td>2475</td>
<td>2025</td>
<td>3025</td>
</tr>
<tr>
<td>70</td>
<td>65</td>
<td>4550</td>
<td>4900</td>
<td>4225</td>
</tr>
<tr>
<td>60</td>
<td>70</td>
<td>4200</td>
<td>3600</td>
<td>4900</td>
</tr>
<tr>
<td>70</td>
<td>90</td>
<td>6300</td>
<td>4900</td>
<td>8100</td>
</tr>
<tr>
<td>75</td>
<td>95</td>
<td>7125</td>
<td>5625</td>
<td>9025</td>
</tr>
<tr>
<td>70</td>
<td>105</td>
<td>7350</td>
<td>4900</td>
<td>11025</td>
</tr>
<tr>
<td>80</td>
<td>110</td>
<td>8800</td>
<td>6400</td>
<td>12100</td>
</tr>
<tr>
<td>95</td>
<td>120</td>
<td>11400</td>
<td>9025</td>
<td>14400</td>
</tr>
<tr>
<td>90</td>
<td>125</td>
<td>11250</td>
<td>8100</td>
<td>15625</td>
</tr>
<tr>
<td>Total: 695</td>
<td>885</td>
<td>65450</td>
<td>51075</td>
<td>84925</td>
</tr>
</tbody>
</table>

The regression line of $Y$ on $X$:

$$Y = a + b X$$

Where

$$a = \frac{(\sum x^2) \cdot (\sum y) - (\sum x) \cdot (\sum xy)}{n \cdot (\sum x^2) - (\sum x)^2}$$

and

$$b = \frac{n \cdot (\sum xy) - (\sum x) \cdot (\sum y)}{n \cdot (\sum x^2) - (\sum x)^2}$$

We obtain

$$a = \frac{(51075 \times 885 - 695 \times 65450)}{(10 \times 51075 - 695^2)}$$

$$= \frac{(45201375 - 45487750)}{(510750 - 483025)}$$

$$= - \frac{286375}{27725}$$

$$= - 10.329$$

$$b = \frac{(10 \times 65450 - 695 \times 885)}{27725}$$

$$= \frac{(654500 - 615075)}{27725}$$

$$= 39425 / 27725$$

$$= 1.422$$

156
So, the regression equation is $Y = -10.329 + 1.422X$

Next, if we consider the regression line of $X$ on $Y$,
We get the equation $X = a + bY$ where

$$a = \frac{\left(\sum y^2\right) (\sum x) - (\sum y) (\sum x y)}{n (\sum y^2) - (\sum y)^2}$$

And

$$b = \frac{n (\sum x y) - (\sum x) (\sum y)}{n (\sum y^2) - (\sum y)^2}.$$  

We get

$$a = \frac{(84925 \times 695 - 885 \times 65450)}{(10 \times 84925 - 885^2)}$$

$$= \frac{(59022875 - 57923250)}{(849250 - 783225)}$$

$$= \frac{1099625}{66025}$$

$$= 16.655,$$

$$b = \frac{10 \times 65450 - 695 \times 885}{66025}$$

$$= \frac{(654500 - 615075)}{66025}$$

$$= \frac{39425}{66025}$$

$$= 0.597$$

So, the regression equation is $X = 16.655 + 0.597Y$

Note:

For the data given in this problem, if we use the formula for $r$, we get

$$r = \frac{N \sum XY - (\sum X)(\sum Y)}{\sqrt{N \sum X^2 - (\sum X)^2} \sqrt{N \sum Y^2 - (\sum Y)^2}}$$
\[= (10 \times 65450 - 695 \times 885) / \left\{ \sqrt{(10 \times 51075 - 695^2)} \right\} \sqrt{(10 \times 84925 - 885^2)}\]

\[= (654500 - 615075) / \left(\sqrt{27725} \sqrt{66025}\right)\]

\[= 39425 / 166.508 \times 256.95\]

\[= 39425 / 42784.23\]

\[= 0.9214\]

However, once we know the two \(b\) values, we can find the coefficient of correlation \(r\) between \(X\) and \(Y\) as the square root of the product of the two \(b\) values. Thus we obtain

\[r = \sqrt{(1.422 \times 0.597)}\]

\[= \sqrt{0.848934}\]

\[= 0.9214.\]

Note that this agrees with the above value of \(r\).

**QUESTIONS**

1. Explain the aim of ‘correlation analysis’.
2. Distinguish between positive and negative correlation.
3. State the formula for simple correlation coefficient.
4. State the properties of the correlation coefficient.
5. What is ‘rank correlation’? Explain.
6. State the formula for rank correlation coefficient.
7. Explain how to resolve ties while calculating ranks.
8. Explain the concept of regression.
10. Explain normal equations in the context of regression analysis.
11. State the formulae for the constant term and coefficient in the regression equation.
12. State the relationship between the regression coefficient and correlation coefficient.
13. Explain the managerial uses of correlation analysis and regression analysis.
2. Analysis Of Variance

Lesson outline

- Definition Of Anova
- Assumptions Of Anova
- Classification Of Linear Models
- Anova For One-Way Classified Data
- Anova Table For One-Way Classified Data
- Null And Alternative Hypotheses
- Type I Error
- Level Of Significance
- SS, Mss And Variance Ratio
- Calculation Of F Value
- Table Value Of F
- Coding Method
- Inference From Anova Table
- Managerial Applications Of Anova

Learning Objectives

After reading this lesson you should be able to

- Understand the Concept of ANOVA
- Formulate Null and Alternative Hypotheses
- Construct ANOVA Table for One-Way Classified Data
- Calculate T, N and CF
- Calculate SS, DF and MSS
- Calculate F Value
- Find the Table Value of F
- Draw Inference From ANOVA
- Apply Coding Method
- Understand the Managerial Applications of ANOVA
ANALYSIS OF VARIANCE (ANOVA)

Introduction

For managerial decision making, sometimes one has to carry out tests of significance. The analysis of variance is an effective tool for this purpose. The objective of the analysis of variance is to test the homogeneity of the means of different samples.

Definition

According to R.A. Fisher, “analysis of variance is the separation of variance ascribable to one group of causes from the variance ascribable to other groups”.

Assumptions of ANOVA

The technique of ANOVA is mainly used for the analysis and interpretation of data obtained from experiments. This technique is based on three important assumptions, namely

1. The parent population is normal.
2. The error component is distributed normally with zero mean and constant variance.
3. The various effects are additive in nature.

The technique of ANOVA essentially consists of partitioning the total variation in an experiment into components of different sources of variation. These sources of variations are due to controlled factors and uncontrolled factors. Since the variation in the sample data is characterized by means of many components of variation, it can be symbolically represented in the mathematical form called a linear model for the sample data.
Classification of models

Linear models for the sample data may broadly be classified into three types as follows:

1. Random effect model
2. Fixed effect model
3. Mixed effect model

In any variance components model, the error component has always random effects, since it occurs purely in a random manner. All other components may be either mixed or random.

Random effect model

A model in which each of the factors has random effect (including error effect) is called a random effect model or simply a random model.

Fixed effect model

A model in which each of the factors has fixed effects, but only the error effect is random is called a fixed effect model or simply a fixed model.

Mixed effect model

A model in which some of the factors have fixed effects and some others have random effects is called a mixed effect model or simply a mixed model.

In what follows, we shall restrict ourselves to a fixed effect model. In a fixed effect model, the main objective is to estimate the effects and find the measure of variability among each of the factors and finally to find the variability among the error effects.

The ANOVA technique is mainly based on the linear model which depends on the types of data used in the linear model. There are several types of data in ANOVA, depending on the number of sources of variation namely,
One-way classified data,

Two-way classified data, ...

m-way classified data.

**One-way classified data**

When the set of observations is distributed over different levels of a single factor, then it gives one-way classified data.

**ANOVA for One-way classified data**

Let $y$ denote the $j^{th}$ observation corresponding to the $i^{th}$ level of factor $A$ and $Y_{ij}$ the corresponding random variate.

Define the linear model for the sample data obtained from the experiment by the equation

$$ y_{ij} = \mu + a_i + e_{ij} \left( i = 1, 2, \ldots, k \right) \left( j = 1, 2, \ldots, n_i \right) $$

Where $\mu$ represents the general mean effect which is fixed and which represents the general condition of the experimental units, $a_i$ denotes the fixed effect due to $i^{th}$ level of the factor $A$ ($i=1,2,\ldots,k$) and hence the variation due to $a_i$ ($i=1,2,\ldots,k$) is said to be control. The last component of the model $e_{ij}$ is the random variable. It is called the error component and it makes the $Y_{ij}$ a random variate. The variation in $e_{ij}$ is due to all the uncontrolled factors and $e_{ij}$ is independently, identically and normally distributed with mean zero and constant variance $\sigma^2$.

For the realization of the random variate $Y_{ij}$, consider $y_{ij}$ defined by

$$ y_{ij} = \mu + a_i + e_{ij} \left( i = 1, 2, \ldots, k \right) \left( j = 1, 2, \ldots, n_i \right) $$
The expected value of the general observation \( y_{ij} \) in the experimental units is given by

\[
E(y_{ij}) = \mu_i \text{ for all } i = 1, 2, ..., k
\]

With \( y_{ij} = \mu_i + e_{ij} \), where \( e_{ij} \) is the random error effect due to uncontrolled factors (i.e., due to chance only).

Here we may expect \( \mu_i = \mu \) for all \( i = 1, 2, ..., k \), if there is no variation due to control factors. If it is not the case, we have

\[
\mu_i \neq \mu \text{ for all } i = 1, 2, ..., k
\]

i.e., \( \mu_i - \mu \neq 0 \) for all \( i = 1, 2, ..., k \)

Suppose \( \mu_i - \mu = a_i \).

Then we have \( \mu_i \neq \mu + a_i \) for all \( i = 1, 2, ..., k \)

On substitution for \( \mu_i \) in the above equation, the linear model reduces to

\[
y_{ij} = \mu + a_i + e_{ij} \quad (i = 1, 2, ..., k)
\]

(1)

The objective of ANOVA is to test the null hypothesis \( H_o : \mu_i = \mu \text{ for all } i = 1, 2, ..., k \) or \( H_o : a_i = 0 \text{ for all } i = 1, 2, ..., k \). For carrying out this test, we need to estimate the unknown parameters \( \mu \) and \( a_i \) for all \( i = 1, 2, ..., k \), by the principle of least squares. This can be done by minimizing the residual sum of squares defined by

\[
E = \sum_{ij} e_{ij}^2 = \sum_{ij} (y_{ij} - \mu - a_i)^2,
\]

Using (1). The normal equations can be obtained by partially differentiating \( E \) with respect to \( \mu \) and \( a_i \) for all \( i = 1, 2, ..., k \) and equating the results to zero. We obtain

\[
G = N\mu + \sum_i n_i a_i
\]  

(2)
and 

\[ T_i = n_i \mu + n_i a_i, \quad i = 1, 2, \ldots, k \]  

(3)

Where \( N = nk \). We see that the number of variables \((k+1)\) is more than the number of independent equations \((k)\). So, by the theorem on a system of linear equations, it follows that unique solution for this system is not possible.

However, by making the assumption that \( \sum_i n_i a_i = 0 \), we can get a unique solution \( \mu \) for \( \mu \) and \( a_i \) \((i = 1, 2, \ldots, k)\). Using this condition in equation (2), we get

\[ G = N \mu \]

i.e. \( \mu = \frac{G}{N} \)

Therefore the estimate of \( \mu \) is given by

\[ \hat{\mu} = \frac{G}{N} \]  

(4)

Again from equation (2), we have

\[ \frac{T_i}{n_i} = \mu + a_i \]

Hence, \( a_i = \frac{T_i}{n_i} - \mu \)

Therefore, the estimate of \( a_i \) is given by

\[ \hat{a}_i = \frac{T_i}{n_i} - \hat{\mu} \]

i.e.,

\[ \hat{a}_i = \frac{T_i}{n_i} - \frac{G}{N} \]  

(5)

Substituting the least square estimates of \( \mu \) and \( \mu_i \) in the residual sum of squares, we get

\[ E = \sum_y (y_{ij} - \hat{\mu} - \hat{\mu}_i)^2 \]
After carrying out some calculations and using the normal equations (2) and (3) we obtain

\[ E = \left( \sum_{ij} y_{ij}^2 - \frac{G^2}{N} \right) - \left( \sum_{i} T_i^2 - \frac{G^2}{N} \right) \]  

(6)

The first term in the RHS of equation (6) is called the corrected total sum of squares while \( \sum_{ij} y_{ij}^2 \) is called the uncorrected total sum of squares.

For measuring the variation due to treatment (controlled factor), we consider the null hypothesis that all the treatment effects are equal.

i.e.,

\[ H_0 : \mu_1 = \mu_2 = \ldots = \mu_k = \mu \]

i.e., \( H_0 : \mu_i = \mu \) for all \( i = 1, 2, \ldots, k \)

i.e., \( H_0 : \mu_i - \mu = 0 \) for all \( i = 1, 2, \ldots, k \)

i.e., \( H_0 : a_i = 0 \)

Under \( H_0 \), the linear model reduces to

\[ y_{ij} = \mu + e_{ij} \left\{ \begin{array}{c} i = 1, 2, \ldots, k \vspace{1mm} \hfill \end{array} \right. \left\{ j = 1, 2, \ldots, n_i \right. \}

Proceeding as before, we get the residual sum of squares for this hypothetical model as

\[ E_1 = \left( \sum_{ij} y_{ij}^2 \right) - \frac{G^2}{N} \]  

(7)

Actually, \( E_1 \) contains the variation due to both treatment and error. Therefore a measure of variation due to treatment can be obtained by “\( E_1 - E \)”. Using (6) and (7), we get
The expression in (8) is usually called the **corrected treatment sum** of squares while the term \( \sum_{i=1}^{k} \frac{T_i^2}{n_i} \) is called **uncorrected treatment sum** of squares. Here it may be noted that \( \frac{G^2}{N} \) is a correction factor (also called a correction term). Since \( E \) is based on \( N - K \) free observations, it has \( N - K \) degrees of freedom (df). Similarly, since \( E_1 \) is based on \( N - 1 \) free observation, \( E_1 \) has \( N - 1 \) degrees of freedom. So \( E_1 - E \) has \( K - 1 \) degrees of freedom.

When actually the null hypothesis is true, if we reject it on the basis of the estimated value in our statistical analysis, we will be committing **Type – I Error**. The probability for committing this error is referred to as the denoted by \( \alpha \). The testing of the null hypothesis \( H_0 \) may be carried out by F test. For given \( \alpha \), we have

\[
F = \frac{\text{TrMSS}}{\text{EMSS}} = \frac{\text{Trss}}{\text{Ess}} \frac{dF}{dF} ; \quad F_{k-1,N-k}
\]

i.e., It follows F distribution with degrees of freedom K-1 and N-K.

All these values are represented in the form of a table called **ANOVA table**, furnished below.
# ANOVA table for one-way classified data

<table>
<thead>
<tr>
<th>Source of Variation</th>
<th>Degrees of Freedom</th>
<th>Sum of Squares (SS)</th>
<th>Mean Squares (MS)</th>
<th>Variance ratio F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Between the level of the factor (Treatment)</td>
<td>k-1</td>
<td>$E_1 - E = Q_T$</td>
<td>$M_T = \frac{Q_T}{k-1}$</td>
<td>$F_T = \frac{M_T}{M_E}$; $F_{k-1,N-k}$</td>
</tr>
<tr>
<td>Within the level of factor (error)</td>
<td>N-k</td>
<td>$Q_E$</td>
<td>$M_E = \frac{Q_E}{N-k}$</td>
<td>-</td>
</tr>
<tr>
<td>Total</td>
<td>N-1</td>
<td>$Q = \sum y_{ij} - \frac{G^2}{N}$</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>
Variance ratio

The variance ratio is the ratio of the greater variance to the smaller variance. It is also called the F-coefficient. We have

\[ F = \frac{\text{greater variance}}{\text{smaller variance}}. \]

We refer to the table of F values at a desired level of significance \( \alpha \). In general, \( \alpha \) is taken to be 5 %. The table value is referred to as the theoretical value or the expected value. The calculated value is referred to as the observed value.

Inference

If the observed value of F is less than the expected value of F (i.e., \( F_o < F_e \)) for the given level of significance \( \alpha \), then the null hypothesis \( H_o \) is accepted. In this case, we conclude that there is no significant difference between the treatment effects.

On the other hand, if the observed value of F is greater than the expected value of F (i.e., \( F_o > F_e \)) for the given level of significance \( \alpha \), then the null hypothesis \( H_o \) is rejected. In this case, we conclude that all the treatment effects are not equal.

Note:

If the calculated value of F and the table value of \( f \) are equal, we can try some other value of \( \alpha \).

Problem 1

The following are the details of sales effected by three sales persons in three door-to-door campaigns.

<table>
<thead>
<tr>
<th>Sales person</th>
<th>Sales in door – to – door campaign</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>8</td>
</tr>
<tr>
<td>B</td>
<td>7</td>
</tr>
<tr>
<td>C</td>
<td>6</td>
</tr>
</tbody>
</table>
Construct an ANOVA table and find out whether there is any significant difference in the performance of the sales persons.

Solution:

Method I (Direct method):

\[
\begin{align*}
\sum A &= 8 + 9 + 5 + 10 = 32 \\
\sum B &= 7 + 6 + 6 + 9 = 28 \\
\sum C &= 6 + 6 + 7 + 5 = 24 \\
\end{align*}
\]

Sample mean for A : \( \bar{A} = \frac{32}{4} = 8 \)

Sample mean for B : \( \bar{B} = \frac{28}{4} = 7 \)

Sample mean for C : \( \bar{C} = \frac{24}{4} = 6 \)

Total number of sample items = No. of items for A + No. of items for B + No. of items for C

= 4 + 4 + 4 = 12

Mean of all the samples \( \bar{X} = \frac{32 + 28 + 24}{12} = \frac{84}{12} = 7 \)
Sum of squares of deviations for A:

<table>
<thead>
<tr>
<th></th>
<th>$A - \bar{A} = A - 8$</th>
<th>$(A - \bar{A})^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>9</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>5</td>
<td>-3</td>
<td>9</td>
</tr>
<tr>
<td>10</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td></td>
<td>14</td>
</tr>
</tbody>
</table>

Sum of squares of deviations for B:

<table>
<thead>
<tr>
<th></th>
<th>$B - \bar{B} = B - 7$</th>
<th>$(B - \bar{B})^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>6</td>
<td>-1</td>
<td>1</td>
</tr>
<tr>
<td>6</td>
<td>-1</td>
<td>1</td>
</tr>
<tr>
<td>9</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td></td>
<td>6</td>
</tr>
</tbody>
</table>

Sum of squares of deviations for C:

<table>
<thead>
<tr>
<th></th>
<th>$C - \bar{C} = C - 6$</th>
<th>$(C - \bar{C})^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>6</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>7</td>
<td>-1</td>
<td>1</td>
</tr>
<tr>
<td>5</td>
<td>-1</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2</td>
</tr>
</tbody>
</table>
Sum of squares of deviations within

\[ \text{Varieties} = \sum(A - \bar{A})^2 + \sum(B - \bar{B})^2 + \sum(C - \bar{C})^2 \]

\[ = 14 + 6 + 2 \]

\[ = 22 \]

Sum of squares of deviations for total variance:

<table>
<thead>
<tr>
<th>Sales person</th>
<th>sales</th>
<th>Sales - ( \bar{X} = \text{sales} - 7 )</th>
<th>((Sales - 7)^2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>8</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>A</td>
<td>9</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>A</td>
<td>5</td>
<td>-2</td>
<td>4</td>
</tr>
<tr>
<td>A</td>
<td>10</td>
<td>3</td>
<td>9</td>
</tr>
<tr>
<td>B</td>
<td>7</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>B</td>
<td>6</td>
<td>-1</td>
<td>1</td>
</tr>
<tr>
<td>B</td>
<td>6</td>
<td>-1</td>
<td>1</td>
</tr>
<tr>
<td>B</td>
<td>9</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>C</td>
<td>6</td>
<td>-1</td>
<td>1</td>
</tr>
<tr>
<td>C</td>
<td>6</td>
<td>-1</td>
<td>1</td>
</tr>
<tr>
<td>C</td>
<td>7</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>C</td>
<td>5</td>
<td>2</td>
<td>4</td>
</tr>
</tbody>
</table>

\[\sum = 30\]
ANOVA Table

<table>
<thead>
<tr>
<th>Source of variation</th>
<th>Degrees of freedom</th>
<th>Sum of squares of deviations</th>
<th>Variance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Between varieties</td>
<td>$3 - 1 = 2$</td>
<td>$8$</td>
<td>$\frac{8}{2} = 4$</td>
</tr>
<tr>
<td>Within varieties</td>
<td>$12 - 3 = 9$</td>
<td>$22$</td>
<td>$\frac{22}{9} = 2.44$</td>
</tr>
<tr>
<td>Total</td>
<td>$12 - 1 = 11$</td>
<td>$30$</td>
<td></td>
</tr>
</tbody>
</table>

Calculation of F value:

$$F = \frac{\text{Greater Variance}}{\text{Smaller Variance}} = \frac{4.00}{2.44} = 1.6393$$

Degrees of freedom for greater variance $(df_g) = 2$

Degrees of freedom for smaller variance $(df_s) = 9$

Let us take the level of significance as 5%

The table value of F = 4.26

Inference:

The calculated value of F is less than the table value of F. Therefore, the null hypothesis is accepted. It is concluded that there is no significant difference in the performance of the sales persons, at 5% level of significance.

Method II (Short cut Method):

$$\Sigma A = 32, \Sigma B = 28, \Sigma C = 24.$$  

$T = \text{Sum of all the sample items}$

172
\[ N = \text{Total number of items in all the samples} = 4 + 4 + 4 = 12 \]

Correction Factor = \[ \frac{T^2}{N} = \frac{84^2}{12} = 588 \]

Calculate the sum of squares of the observed values as follows:

<table>
<thead>
<tr>
<th>Sales Person</th>
<th>X</th>
<th>X^2</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>8</td>
<td>64</td>
</tr>
<tr>
<td>A</td>
<td>9</td>
<td>81</td>
</tr>
<tr>
<td>A</td>
<td>5</td>
<td>25</td>
</tr>
<tr>
<td>A</td>
<td>10</td>
<td>100</td>
</tr>
<tr>
<td>B</td>
<td>7</td>
<td>49</td>
</tr>
<tr>
<td>B</td>
<td>6</td>
<td>36</td>
</tr>
<tr>
<td>B</td>
<td>6</td>
<td>36</td>
</tr>
<tr>
<td>B</td>
<td>9</td>
<td>81</td>
</tr>
<tr>
<td>C</td>
<td>6</td>
<td>36</td>
</tr>
<tr>
<td>C</td>
<td>6</td>
<td>36</td>
</tr>
<tr>
<td>C</td>
<td>7</td>
<td>49</td>
</tr>
<tr>
<td>C</td>
<td>5</td>
<td>25</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(\sum X^2)</td>
</tr>
</tbody>
</table>

\[ \text{Sum of squares of deviations for total variance} = \sum X^2 - \text{correction factor} \]

\[ = 618 - 588 = 30. \]
Sum of squares of deviations for variance between samples

\[
\sum_{A}^2 + \sum_{B}^2 + \sum_{C}^2 - CF
\]

\[
= \frac{32^2}{4} + \frac{28^2}{4} + \frac{24^2}{4} - 588
\]

\[
= \frac{1024}{4} + \frac{784}{4} + \frac{576}{4} - 588
\]

\[
= 256 + 196 + 144 - 588
\]

\[
= 8
\]

**ANOVA Table**

<table>
<thead>
<tr>
<th>Source of variation</th>
<th>Degrees of freedom</th>
<th>Sum of squares of deviations</th>
<th>Variance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Between varieties</td>
<td>3-1 = 2</td>
<td>8</td>
<td>(\frac{8}{2} = 4)</td>
</tr>
<tr>
<td>Within varieties</td>
<td>12 - 3 = 9</td>
<td>22</td>
<td>(\frac{22}{9} = 2.44)</td>
</tr>
<tr>
<td>Total</td>
<td>12 - 1 = 11</td>
<td>30</td>
<td></td>
</tr>
</tbody>
</table>

It is to be noted that the ANOVA tables in the methods I and II are one and the same. For the further steps of calculation of F value and drawing inference, refer to method I.
Problem 2

The following are the details of plinth areas of ownership apartment flats offered by 3 housing companies A, B, C. Use analysis of variance to determine whether there is any significant difference in the plinth areas of the apartment flats.

<table>
<thead>
<tr>
<th>Housing Company</th>
<th>Plinth area of apartment flats</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1500 1430 1550 1450</td>
</tr>
<tr>
<td>B</td>
<td>1450 1550 1600 1480</td>
</tr>
<tr>
<td>C</td>
<td>1550 1420 1450 1430</td>
</tr>
</tbody>
</table>

Use analysis of variance to determine whether there is any significant difference in the plinth areas of the apartment's flats.

Note:

As the given figures are large, working with them will be difficult. Therefore, we use the following facts:

i). Variance ratio is independent of the change of origin.

ii.) Variance ratio is independent of the change of scale.

In the problem under consideration, the numbers vary from 1420 to 1600. So we follow a method called the coding method. First, let us subtract 1400 from each item. We get the following transformed data:

<table>
<thead>
<tr>
<th>Company</th>
<th>Transformed measurement</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>100 30 150 50</td>
</tr>
<tr>
<td>B</td>
<td>50 150 100 80</td>
</tr>
<tr>
<td>C</td>
<td>150 20 50 30</td>
</tr>
</tbody>
</table>

Next, divide each entry by 10.
The transformed data are given below.

<table>
<thead>
<tr>
<th>Company</th>
<th>Transformed measurement</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>10 3 15 5</td>
</tr>
<tr>
<td>B</td>
<td>5 15 10 8</td>
</tr>
<tr>
<td>C</td>
<td>15 2 5 3</td>
</tr>
</tbody>
</table>

We work with these transformed data. We have

\[
\sum A = 10 + 3 + 15 + 5 = 33 \\
\sum B = 5 + 15 + 10 + 8 = 38 \\
\sum C = 15 + 2 + 5 + 3 = 25 \\
\sum T = \sum A + \sum B + \sum C \\
= 33 + 38 + 25 \\
= 96
\]

\[N = \text{Total number of items in all the samples} = 4 + 4 + 4 = 12\]

\[
\text{Correction factor} = \frac{T^2}{N} = \frac{96^2}{12} = 768
\]

Calculate the sum of squares of the observed values as follows:

<table>
<thead>
<tr>
<th>Company</th>
<th>X</th>
<th>X^2</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>10</td>
<td>100</td>
</tr>
<tr>
<td>A</td>
<td>3</td>
<td>9</td>
</tr>
<tr>
<td>A</td>
<td>15</td>
<td>225</td>
</tr>
<tr>
<td>A</td>
<td>5</td>
<td>25</td>
</tr>
<tr>
<td>B</td>
<td>5</td>
<td>25</td>
</tr>
<tr>
<td>B</td>
<td>15</td>
<td>225</td>
</tr>
<tr>
<td>B</td>
<td>10</td>
<td>100</td>
</tr>
<tr>
<td>B</td>
<td>8</td>
<td>64</td>
</tr>
<tr>
<td>C</td>
<td>15</td>
<td>225</td>
</tr>
</tbody>
</table>
Sum of squares of deviations for total variance = $\sum X^2$ - correction factor

= 1036 – 768 = 268

Sum of squares of deviations for variance between samples

$$\sum \sum \sum \frac{A^2}{N_1} + \frac{B^2}{N_2} + \frac{C^2}{N_3} - CF$$

$$= \frac{33^2}{4} + \frac{38^2}{4} + \frac{25^2}{4} - 768$$

$$= \frac{1089}{4} + \frac{1444}{4} + \frac{625}{4} - 768$$

$$= 272.25 + 361 + 156.25 - 768$$

$$= 789.5 - 768$$

$$= 21.5$$

ANOVA Table

<table>
<thead>
<tr>
<th>Source of variation</th>
<th>Degrees of freedom</th>
<th>Sum of squares of deviation</th>
<th>Variance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Between varieties</td>
<td>3-1 = 2</td>
<td>21.5</td>
<td>$\frac{21.5}{2} = 10.75$</td>
</tr>
<tr>
<td>Within varieties</td>
<td>12 – 3 = 9</td>
<td>264.5</td>
<td>$\frac{24.65}{9} = 27.38$</td>
</tr>
<tr>
<td>Total</td>
<td>12 – 1 = 11</td>
<td>268</td>
<td></td>
</tr>
</tbody>
</table>
Calculation of F value:

\[
F = \frac{\text{Greater Variance}}{\text{Smaller Variance}} = \frac{27.38}{10.75} = 2.5470
\]

Degrees of freedom for greater variance \((df_1) = 9\)

Degrees of freedom for smaller variance \((df_2) = 2\)

The table value of f at 5% level of significance = 19.38

Inference:

Since the calculated value of F is less than the table value of F, the null hypothesis is accepted and it is concluded that there is no significant difference in the plinth areas of ownership apartment flats offered by the three companies, at 5% level of significance.

Problem 3

A finance manager has collected the following information on the performance of three financial schemes.

<table>
<thead>
<tr>
<th>Source of variation</th>
<th>Degrees of freedom</th>
<th>Sum of squares of deviations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Treatments</td>
<td>5</td>
<td>15</td>
</tr>
<tr>
<td>Residual</td>
<td>2</td>
<td>25</td>
</tr>
<tr>
<td>Total (corrected)</td>
<td>7</td>
<td>40</td>
</tr>
</tbody>
</table>

Interpret the information obtained by him.

Note: ‘Treatments’ means ‘Between varieties’. ‘Residual’ means ‘Within varieties’ or ‘Error’. 
Solution:

Number of schemes = 3 \ (since \ 3 - 1 = 2)

Total number of sample items = 8 \ (since \ 8 - 1 = 7)
Let us calculate the variance.

Variance between varieties = \frac{15}{2} = 7.5

Variance between varieties = \frac{25}{5} = 5

\[ F = \frac{Greater\ Variance}{Smaller\ Variance} = \frac{7.5}{5} = 1.5 \]

Degrees of freedom for greater variance

\[ (df_i) = 2 \]

Degrees of freedom for smaller variance

\[ (df_i) = 5 \]

The total value of F at 5% level of significance

= 5.79

Inference:

Since the calculated value of F is less than the table value of F we accept the null-hypothesis and conclude that there is no significant difference in the performance of the three financial schemes.
QUESTIONS

1. Define analysis of variance.
2. State the assumptions in analysis of variance.
3. Explain the classification of linear models for the sample data.
4. Explain anova table.
5. Explain how inference is drawn from anova table.
6. Explain the managerial applications of analysis of variance.

***
UNIT IV

3. Partial And Multiple Correlation

Lesson Outline

› The Concept Of Partial Correlation
› The Concept Of Multiple Correlation

Learning Objectives

After reading this lesson you should be able to

› Determine Partial Correlation Coefficient
› Determine Multiple Correlation Coefficient

I. PARTIAL CORRELATION

Simple correlation is a measure of the relationship between a dependent variable and another independent variable. For example, if the performance of a sales person depends only on the training that he has received, then the relationship between the training and the sales performance is measured by the simple correlation coefficient $r$. However, a dependent variable may depend on several variables. For example, the yarn produced in a factory may depend on the efficiency of the machine, the quality of cotton, the efficiency of workers, etc. It becomes necessary to have a measure of relationship in such complex situations. Partial correlation is used for this purpose. The technique of partial correlation proves useful when one has to develop a model with 3 to 5 variables.

Suppose $Y$ is a dependent variable, depending on $n$ other variables $X_1, X_2, \ldots, X_n$. Partial correlation is a measure of the relationship between $Y$ and any one of the variables $X_1, X_2, \ldots, X_n$, as if the other variables have been eliminated from the situation.
The partial correlation coefficient is defined in terms of simple correlation coefficients as follows:

Let $r_{12.3}$ denote the correlation of $X_1$ and $X_2$ by eliminating the effect of $X_3$.

Let $r_{12}$ be the simple correlation coefficient between $X_1$ and $X_2$.

Let $r_{13}$ be the simple correlation coefficient between $X_1$ and $X_3$.

Let $r_{23}$ be the simple correlation coefficient between $X_2$ and $X_3$.

Then we have

$$r_{12.3} = \frac{r_{12} - r_{13} r_{23}}{\sqrt{(1 - r_{13}^2)(1 - r_{23}^2)}}$$

Similarly,

$$r_{13.2} = \frac{r_{13} - r_{12} r_{32}}{\sqrt{(1 - r_{12}^2)(1 - r_{32}^2)}}$$

and

$$r_{23.1} = \frac{r_{23} - r_{21} r_{13}}{\sqrt{(1 - r_{21}^2)(1 - r_{13}^2)}}$$
Problem 1

Given that $r_{12} = 0.6$, $r_{13} = 0.58$, $r_{23} = 0.70$ determine the partial correlation coefficient $r_{12.3}$

Solution:

We have

$$r_{12.3} = \frac{0.6 - 0.58 \times 0.70}{\sqrt{(1 - (0.58)^2) (1 - (0.70)^2)}}$$

$$= \frac{0.6 - 0.406}{\sqrt{(1 - 0.3364) (1 - 0.49)}}$$

$$= \frac{0.194}{\sqrt{0.6636 \times 0.51}}$$

$$= \frac{0.194}{0.8146 \times 0.7141}$$

$$= \frac{0.194}{0.5817}$$

$$= 0.3335$$
Problem 2

If \( r_{12} = 0.75, \ r_{13} = 0.80, \ r_{23} = 0.70 \), find the partial correlation coefficient \( r_{13.2} \)

Solution:

We have

\[
\begin{align*}
r_{13.2} &= \frac{r_{13} - r_{12} \cdot r_{32}}{\sqrt{(1 - r_{12}^2) \cdot (1 - r_{32}^2)}} \\
&= \frac{0.8 - 0.75 \cdot 0.70}{\sqrt{(1 - 0.75^2) \cdot (1 - 0.70^2)}} \\
&= \frac{0.8 - 0.525}{\sqrt{(1 - 0.5625) \cdot (1 - 0.49)}} \\
&= \frac{0.275}{\sqrt{0.4375} \cdot 0.51} \\
&= \frac{0.275}{0.6614 \cdot 0.7141} \\
&= \frac{0.275}{0.4723} \\
&= 0.5823
\end{align*}
\]

II. MULTIPLE CORRELATION

When the value of a variable is influenced by another variable, the relationship between them is a simple correlation. In a real life situation, a variable may be influenced by many other variables. For example, the sales achieved for a product may depend on the income of the consumers, the price, the quality of the product, sales promotion techniques, the channels of distribution, etc. In this case, we have to consider the joint influence
of several independent variables on the dependent variable. Multiple correlations arise in this context.

Suppose Y is a dependent variable, which is influenced by n other variables \(X_1, X_2, \ldots, X_n\). The multiple correlation is a measure of the relationship between Y and \(X_1, X_2, \ldots, X_n\) considered together.

The multiple correlation coefficients are denoted by the letter R. The dependent variable is denoted by \(X_1\). The independent variables are denoted by \(X_2, X_3, X_4, \ldots\), etc.

**Meaning of notations:**

\[ R_{1,23} \] denotes the multiple correlation of the dependent variable \(X_1\) with two independent variables \(X_2\) and \(X_3\). It is a measure of the relationship that \(X_1\) has with \(X_2\) and \(X_3\).

\[ R_{2,13} \] is the multiple correlation of the dependent variable \(X_2\) with two independent variables \(X_1\) and \(X_3\).

\[ R_{3,12} \] is the multiple correlation of the dependent variable \(X_3\) with two independent variables \(X_1\) and \(X_2\).

\[ R_{1,234} \] is the multiple correlation of the dependent variable \(X_1\) with three independent variables \(X_2, X_3\) and \(X_4\).

**Coefficient Of Multiple Linear Correlations**

The coefficient of multiple linear correlation is given in terms of the partial correlation coefficients as follows:

\[
R_{1,23} = \frac{\sqrt{r_{12}^2 + r_{13}^2 - 2r_{12}r_{13}r_{23}}}{\sqrt{1 - r_{23}^2}}
\]

\[
R_{2,13} = \frac{\sqrt{r_{21}^2 + r_{23}^2 - 2r_{21}r_{23}r_{13}}}{\sqrt{1 - r_{13}^2}}
\]
Properties Of The Coefficient Of Multiple Linear Correlations:

1. The coefficient of multiple linear correlations \( R \) is a non-negative quantity. It varies between 0 and 1.

2. \( R_{1.23} = R_{1.32} \)
   
3. \( R_{1.32} \geq |r_{12}| \),
   
4. \( R_{1.32} \geq |r_{13}| \), etc.

Problem 3

If the simple correlation coefficients have the values \( r_{12} = 0.6, r_{13} = 0.65, r_{23} = 0.8 \), find the multiple correlation coefficient \( R_{1.23} \)

Solution:

We have

\[
R_{1.23} = \frac{\sqrt{r_{12}^2 + r_{13}^2 - 2 r_{12} r_{13} r_{23}}}{\sqrt{1 - r_{23}^2}}
\]

\[
= \frac{\sqrt{(0.6)^2 + (0.65)^2 - 2 \times 0.6 \times 0.65 \times 0.8}}{\sqrt{1 - (0.8)^2}}
\]
\[ R_{2.13} = \sqrt{\frac{r_{21}^2 + r_{23}^2 - 2r_{21}r_{23}r_{13}}{1 - r_{13}^2}} \]

\[ = \sqrt{\frac{(0.7)^2 + (0.85)^2 - 2 \times 0.7 \times 0.85 \times 0.75}{1 - (0.75)^2}} \]

\[ = \sqrt{\frac{0.49 + 0.7225 - 0.8925}{1 - 0.5625}} \]

\[ = \sqrt{\frac{1.2125 - 0.8925}{0.4375}} \]

\[ = \sqrt{\frac{0.32}{0.4375}} \]

\[ = \sqrt{0.7314} \]

\[ = 0.8552 \]
QUESTIONS

1. Explain partial correlation.
2. Explain multiple correlations.
3. State the properties of the coefficient of multiple linear correlations.
UNIT IV

4. Factor Analysis And Conjoint Analysis

Lesson Outline

- Factor Analysis
- Conjoint Analysis
- Steps In Development Of Conjoint Analysis
- Applications Of Conjoint Analysis
- Advantages And Disadvantages Of Conjoint Analysis
- Illustrative Problems
- Multi-Factor Evaluation Approach In Conjoint Analysis
- Two-Factor Evaluation Approach In Conjoint Analysis

Learning Objectives

After reading this lesson you should be able to

- Understand The Concept Of Factor Analysis
- Understand The Managerial Applications Of Factor Analysis
- Understand The Concept Of Conjoint Analysis
- Apply Rating Scale Technique In Conjoint Analysis
- Apply Ranking Method In Conjoint Analysis
- Apply Mini-Max Scaling Method In Conjoint Analysis
- Understand Multi-Factor Evaluation Approach
- Understand Two-Factor Evaluation Approach
- Understand The Managerial Applications Of Conjoint Analysis

PART I - FACTOR ANALYSIS

In a real life situation, several variables are operating. Some variables may be highly correlated among themselves. For example, if manager of a restaurant has to analyse six attributes of a new product. He undertakes a sample survey and finds out the responses of potential consumers. He
obtains the following attribute correlation matrix.

<table>
<thead>
<tr>
<th>Attribute</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.00</td>
<td>0.05</td>
<td>0.10</td>
<td>0.95</td>
<td>0.20</td>
<td>0.02</td>
</tr>
<tr>
<td>2</td>
<td>0.05</td>
<td>1.00</td>
<td>0.15</td>
<td>0.10</td>
<td>0.60</td>
<td>0.85</td>
</tr>
<tr>
<td>3</td>
<td>0.10</td>
<td>0.15</td>
<td>1.00</td>
<td>0.50</td>
<td>0.55</td>
<td>0.10</td>
</tr>
<tr>
<td>4</td>
<td>0.95</td>
<td>0.10</td>
<td>0.50</td>
<td>1.00</td>
<td>0.12</td>
<td>0.08</td>
</tr>
<tr>
<td>5</td>
<td>0.20</td>
<td>0.60</td>
<td>0.55</td>
<td>0.12</td>
<td>1.00</td>
<td>0.80</td>
</tr>
<tr>
<td>6</td>
<td>0.02</td>
<td>0.85</td>
<td>0.10</td>
<td>0.08</td>
<td>0.80</td>
<td>1.00</td>
</tr>
</tbody>
</table>

Attribute Correlation Matrix

We try to group the attributes by their correlations. The high correlation values are observed for the following attributes:

Attributes 1, 4 with a very high correlation coefficient of 0.95.

Attributes 2, 4 with a high correlation coefficient of 0.85.

Attributes 3, 4 with a high correlation coefficient of 0.85.

As a result, it is seen that not all the attributes are independent. The attributes 1 and 4 have mutual influence on each other while the attributes 2, 5 and 6 have mutual influence among themselves. As far as attribute 3 is concerned, it has little correlation with the attributes 1, 2 and 6. Even with the other attributes 4 and 5, its correlation is not high. However, we can say that attribute 3 is somewhat closer to the variables 4 and 5 rather than the attributes 1, 2 and 6. Thus, from the given list of 6 attributes, it is possible to find out 2 or 3 common factors as follows:

I.
1) The common features of the attributes 1, 3, 4 will give a factor
2) The common features of the attributes 2, 5, 6 will give a factor

Or

II.
1) The common features of the attributes 1, 4 will give a factor
2) The common features of the attributes 2, 5, 6 will give a factor
3) the attribute 3 can be considered to be an independent factor
The factor analysis is a multivariate method. It is a statistical technique to identify the underlying factors among a large number of interdependent variables. It seeks to extract common factor variances from a given set of observations. It splits a number of attributes or variables into a smaller group of uncorrelated factors. It determines which variables belong together. This method is suitable for the cases with a number of variables having a high degree of correlation.

In the above example, we would like to filter down the attributes 1, 4 into a single attribute. Also we would like to do the same for the attributes 2, 5, 6. If a set of attributes (variables) A1, A2, ..., Ak filter down to an attribute Ai (1 ≤ i ≤ k), we say that these attributes are loaded on the factor Ai or saturated with the factor Ai. Sometimes, more than one factor also may be identified.

**Basic concepts in factor analysis**

The following are the key concepts on which factor analysis is based.

**Factor:**

A factor plays a fundamental role among a set of attributes or variables. These variables can be filtered down to the factor. A factor represents the combined effect of a set of attributes. Either there may be one such factor or several such factors in a real life problem based on the complexity of the situation and the number of variables operating.

**Factor loading:**

A factor loading is a value that explains how closely the variables are related to the factor. It is the correlation between the factor and the variable. While interpreting a factor, the absolute value of the factor is taken into account.
Communality:

It is a measure of how much each variable is accounted for by the underlying factors together. It is the sum of the squares of the loadings of the variable on the common factors. If A, B, C, ... are the factors, then the communality of a variable is computed using the relation

\[ h^2 = (\text{The factor loading of the variable with respect to factor A})^2 + \]

\[ \quad (\text{The factor loading of the variable with respect to factor B})^2 + \]

\[ \quad (\text{The factor loading of the variable with respect to factor C})^2 + \ldots \]

Eigen value:

The sum of the squared values of factor loadings pertaining to a factor is called an eigen value. It is a measure of the relative importance of each factor under consideration.

Total Sum Of Squares (TSS)

It is the sum of the eigen values of all the factors.

Application Of Factor Analysis:

1. Model Building For New Product Development:

As pointed out earlier, a real life situation is highly complex and it consists of several variables. A model for the real life situation can be built by incorporating as many features of the situation as possible. But then, with a multitude of features, it is very difficult to build such a highly idealistic model. A practical way is to identify the important variables and incorporate them in the model. Factor analysis seeks to identify those variables which are highly correlated among themselves and find a common factor which can be taken as a representative of those variables. Based on the factor loading, some of variables can be merged together to give a common factor and then a model can be built by incorporating such factors. Identification of the most common features of a product preferred by the consumers will be helpful in the development of new products.
2. Model Building For Consumers:

Another application of factor analysis is to carry out a similar exercise for the respondents instead of the variables themselves. Using the factor loading, the respondents in a research survey can be sorted out into various groups in such a way that the respondents in a group have more or less homogeneous opinions on the topics of the survey. Thus a model can be constructed on the groups of consumers. The results emanating from such an exercise will guide the management in evolving appropriate strategies towards market segmentation.

PART II - CONJOINT ANALYSIS

Introduction

Everything in the world is undergoing a change. There is a proverb saying that “the old order changes, yielding place to new”. Due to rapid advancement in science and technology, there is fast communication across the world. Consequently, the whole world has shrunk into something like a village and thus now-a-days one speaks of the “global village”. Under the present set-up, one can purchase any product of his choice from whatever part of the world it may be available. Because of this reason, what was a seller's market a few years back has transformed into a buyer’s market now.

In a seller’s market of yesterday, the manufacturer or the seller could pass on a product according to his own perceptions and prescriptions. In the buyer’s market of today, a buyer decides what he should purchase, what should be the quality of the product, how much to purchase, where to purchase, when to purchase, at what cost to purchase, from whom to purchase, etc. A manager is perplexed at the way a consumer takes a decision on the purchase of a product. In this background, conjoint analysis is an effective tool to understand a buyer's preferences for a good or service.

Meaning Of Conjoint Analysis

A product or service has several attributes. By an attribute, we mean a characteristic, a property, a feature, a quality, a specification or an aspect. A buyer's decision to purchase a good or service is based on
not just one attribute but a combination of several attributes. i.e., he is concerned with a join of attributes.

Therefore, finding out the consumer's preferences for individual attributes of a product or service may not yield accurate results for a marketing research problem. In view of this fact, conjoint analysis seeks to find out the consumer's preferences for a 'join of attributes', i.e., a combination of several attributes.

Let us consider an example. Suppose a consumer desires to purchase a wrist watch. He would take into consideration several attributes of a wrist watch, namely the configuration details such as mechanism, size, dial, appearance, colour and other particulars such as strap, price, durability, warranty, after-sales service, etc. If a consumer is asked what the important aspect among the above list is, he would reply that all attributes are important for him and so a manager cannot arrive at a decision on the design of a wrist watch. Conjoint analysis assumes that the buyer will base his decision not on just the individual attributes of the product but rather he would consider various combinations of the attributes, such as

‘mechanism, colour, price, after-sales service’,

or ‘dial, colour, durability, warranty’,

or ‘dial, appearance, price, durability’, etc.

This analysis would enable a manager in his decision making process in the identification of some of the preferred combinations of the features of the product.

The rank correlation method seeks to assess the consumer's preferences for individual attributes. In contrast, the conjoint analysis seeks to assess the consumer's preferences for combinations (or groups) of attributes of a product or a service. This method is also called an 'unfolding technique' because preferences on groups of attributes unfold from the rankings expressed by the consumers. Another name for this method is 'multi-attribute compositional model' because it deals with combinations of attributes.
Steps In The Development Of Conjoint Analysis

The development of conjoint analysis comprises of the following steps:

1. Collect a list of the attributes (features) of a product or a service.

2. For each attribute, fix a certain number of points or marks. The more the number of points for an attribute, the more serious the consumers’ concern on that attribute.

3. Select a list of combinations of various attributes.

4. Decide a mode of presentation of the attributes to the respondents of the study i.e., whether it should be in written form, or oral form, or a pictorial representation etc.

5. Inform the combinations of the attributes to the prospective customers.

6. Request the respondents to rank the combinations, or to rate them on a suitable scale, or to choose between two different combinations at a time.

7. Decide a procedure to aggregate the responses from the consumers. Any one of the following procedures may be adopted:
   i. Go by the individual responses of the consumers.
   ii. Put all the responses together and construct a single utility function.
   iii. Split the responses into a certain number of segments such that within each segment, the preferences would be similar.

8. Choose the appropriate technique to analyze the data collected from the respondents.

9. Identify the most preferred combination of attributes.

10. Incorporate the result in designing a new product, construction of an advertisement copy, etc.
Applications Of Conjoint Analysis

1. An idea of consumer’s preferences for combinations of attributes will be useful in designing new products or modification of an existing product.

2. A forecast of the profits to be earned by a product or a service.

3. A forecast of the market share for the company’s product.

4. A forecast of the shift in brand loyalty of the consumers.

5. A forecast of differences in responses of various segments of the product.

6. Formulation of marketing strategies for the promotion of the product.

7. Evaluation of the impact of alternative advertising strategies.

8. A forecast of the consumers’ reaction to pricing policies.

9. A forecast of the consumers’ reaction on the channels of distribution.

10. Evolving an appropriate marketing mix.

11. Even though the technique of conjoint analysis was developed for the formulation of corporate strategy, this method can be used to have a comprehensive knowledge of a wide range of areas such as family decision making process, pharmaceuticals, tourism development, public transport system, etc.

Advantages Of Conjoint Analysis

1. The analysis can be carried out on physical variables.

2. Preferences by different individuals can be measured and pooled together to arrive at a decision.

Disadvantages Of Conjoint Analysis

1. When more and more attributes of a product are included in the study, the number of combinations of attributes also increases, rendering the study highly difficult. Consequently, only a few selected attributes can be included in the study.
2. Gathering of information from the respondents will be a tough job.

3. Whenever novel combinations of attributes are included, the respondents will have difficulty in capturing such combinations.

4. The psychological measurements of the respondents may not be accurate. In spite of the above stated disadvantages, conjoint analysis offers more scope to the researchers in identifying the consumers’ preferences for groups of attributes.

**Illustrative Problem 1 : Application Of Rating Scale Technique**

A wrist watch manufacturer desires to find out the combinations of attributes that a consumer would be interested in. After considering several attributes, the manufacturer identifies the following combinations of attributes for carrying out marketing research.

- Combination – I  Mechanism, colour, price, after-scales service
- Combination – II  Dial, colour, durability, warranty
- Combination – III  Dial, appearance, price, durability
- Combination – IV  Mechanism, dial, price, warranty

12 respondents are asked to rate the 4 combinations on the following 3-point rating scale.

- Scale – 1  :  Less important
- Scale – 2  :  Somewhat important
- Scale – 3  :  Very important
Their responses are given in the following table:

<table>
<thead>
<tr>
<th>Respondent No.</th>
<th>Combination I</th>
<th>Combination II</th>
<th>Combination III</th>
<th>Combination IV</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Less important</td>
<td>Some what important</td>
<td>Very important</td>
<td>Some what important</td>
</tr>
<tr>
<td>2</td>
<td>Some what important</td>
<td>Very important</td>
<td>Less important</td>
<td>Some what important</td>
</tr>
<tr>
<td>3</td>
<td>Some what important</td>
<td>Less important</td>
<td>Some what important</td>
<td>Very important</td>
</tr>
<tr>
<td>4</td>
<td>Less important</td>
<td>Less important</td>
<td>Very important</td>
<td>Some what important</td>
</tr>
<tr>
<td>5</td>
<td>Some what important</td>
<td>Very important</td>
<td>Very important</td>
<td>Less important</td>
</tr>
<tr>
<td>6</td>
<td>Some what important</td>
<td>Very important</td>
<td>Some what important</td>
<td>Less important</td>
</tr>
<tr>
<td>7</td>
<td>Some what important</td>
<td>Less important</td>
<td>Very important</td>
<td>Less important</td>
</tr>
<tr>
<td>8</td>
<td>Very important</td>
<td>Some what important</td>
<td>Less important</td>
<td>Some what important</td>
</tr>
<tr>
<td>9</td>
<td>Very important</td>
<td>Less important</td>
<td>Some what important</td>
<td>Some what important</td>
</tr>
<tr>
<td>10</td>
<td>Some what important</td>
<td>Very important</td>
<td>Less important</td>
<td>Some what important</td>
</tr>
<tr>
<td>11</td>
<td>Very important</td>
<td>Some what important</td>
<td>Very important</td>
<td>Some what important</td>
</tr>
</tbody>
</table>
Determine the most important and the least important combinations of the attributes.

**Solution:**

Let us assign scores to the scales as follows:

<table>
<thead>
<tr>
<th>Sl. No.</th>
<th>Scale</th>
<th>Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Less important</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>Some what important</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>Very important</td>
<td>5</td>
</tr>
</tbody>
</table>

The scores for the four combinations are calculated as follows:

<table>
<thead>
<tr>
<th>Combination</th>
<th>Response</th>
<th>Score for Response</th>
<th>No. Of Respondents</th>
<th>Total score</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>Less important</td>
<td>1</td>
<td>2</td>
<td>1X2=2</td>
</tr>
<tr>
<td></td>
<td>Some what important</td>
<td>3</td>
<td>6</td>
<td>3X6=18</td>
</tr>
<tr>
<td></td>
<td>Very important</td>
<td>5</td>
<td>4</td>
<td>5X4=20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>12</td>
</tr>
<tr>
<td>II</td>
<td>Less important</td>
<td>1</td>
<td>5</td>
<td>1X5=5</td>
</tr>
<tr>
<td></td>
<td>Some what important</td>
<td>3</td>
<td>3</td>
<td>3X3=9</td>
</tr>
<tr>
<td></td>
<td>Very important</td>
<td>5</td>
<td>4</td>
<td>5X4=20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>12</td>
</tr>
<tr>
<td>III</td>
<td>Less important</td>
<td>1</td>
<td>3</td>
<td>1 X 3 = 3</td>
</tr>
<tr>
<td></td>
<td>Some what important</td>
<td>3</td>
<td>3</td>
<td>3 X 3 = 9</td>
</tr>
<tr>
<td></td>
<td>Very important</td>
<td>5</td>
<td>6</td>
<td>5 X 6 = 30</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>12</td>
</tr>
</tbody>
</table>
Let us tabulate the scores earned by the four combinations as follows:

<table>
<thead>
<tr>
<th>Combination</th>
<th>Total scores</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>40</td>
</tr>
<tr>
<td>II</td>
<td>34</td>
</tr>
<tr>
<td>III</td>
<td>42</td>
</tr>
<tr>
<td>IV</td>
<td>32</td>
</tr>
</tbody>
</table>

Inference:

It is concluded that the consumers consider combination III as the most important and combination IV as the least important.

Note:

For illustrating the concepts involved, we have taken up 12 respondents in the above problem. In actual research work, we should take a large number of respondents, say 200 or 100. In any case, the number of respondents shall not be less than 30.

**Illustrative Problem 2: Application Of Ranking Method**

A marketing manager selects four combinations of features of a product for study. The following are the ranks awarded by 10 respondents. Rank one means the most important and rank 4 means the least important.
Determine the most important and the least important combinations of the features of the product.

**Solution:**

Let us assign scores to the ranks as follows:

<table>
<thead>
<tr>
<th>Rank</th>
<th>Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10</td>
</tr>
<tr>
<td>2</td>
<td>8</td>
</tr>
<tr>
<td>3</td>
<td>6</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
</tr>
</tbody>
</table>

The scores for the 4 combinations are calculated as follows:

<table>
<thead>
<tr>
<th>Combination</th>
<th>Rank</th>
<th>Score for rank</th>
<th>No. of respondents</th>
<th>Total score</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>1</td>
<td>10</td>
<td>3</td>
<td>10 X 3 = 30</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>8</td>
<td>1</td>
<td>8 X 1 = 8</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>6</td>
<td>3</td>
<td>6 X 3 = 18</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>4</td>
<td>3</td>
<td>4 X 3 = 12</td>
</tr>
<tr>
<td>II</td>
<td></td>
<td></td>
<td></td>
<td>68</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>10</td>
<td>5</td>
<td>10 X 5 = 50</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>8</td>
<td>3</td>
<td>8 X 3 = 24</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>6</td>
<td>1</td>
<td>6 X 1 = 6</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>4</td>
<td>1</td>
<td>4 X 1 = 4</td>
</tr>
</tbody>
</table>
The final scores for the 4 combinations are as follows:

<table>
<thead>
<tr>
<th>Combination</th>
<th>Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>68</td>
</tr>
<tr>
<td>II</td>
<td>84</td>
</tr>
<tr>
<td>III</td>
<td>66</td>
</tr>
<tr>
<td>IV</td>
<td>62</td>
</tr>
</tbody>
</table>

**Inference:**

It is seen that combination II is the most preferred one by the consumers and combination IV is the least preferred one.

**Illustrative Problem 3:**

**Application Of Mini-Max Scaling Method**

An insurance manager chooses 5 combinations of attributes of a social security plan for analysis. He requests 10 respondents to indicate their perceptions on the importance of the combinations by awarding
the minimum score and the maximum score for each combination in the range of 0 to 100. The details of the responses are given below. Help the manager in the identification of the most important and the least important combinations of the attributes of the social security plan.

<table>
<thead>
<tr>
<th>Respondent No.</th>
<th>Combination I</th>
<th>Combination II</th>
<th>Combination III</th>
<th>Combination IV</th>
<th>Combination V</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Min</td>
<td>Max</td>
<td>Min</td>
<td>Max</td>
<td>Min</td>
</tr>
<tr>
<td>1</td>
<td>30</td>
<td>60</td>
<td>45</td>
<td>85</td>
<td>50</td>
</tr>
<tr>
<td>2</td>
<td>35</td>
<td>65</td>
<td>50</td>
<td>80</td>
<td>50</td>
</tr>
<tr>
<td>3</td>
<td>40</td>
<td>70</td>
<td>35</td>
<td>80</td>
<td>60</td>
</tr>
<tr>
<td>4</td>
<td>40</td>
<td>80</td>
<td>40</td>
<td>80</td>
<td>50</td>
</tr>
<tr>
<td>5</td>
<td>30</td>
<td>75</td>
<td>50</td>
<td>80</td>
<td>60</td>
</tr>
<tr>
<td>6</td>
<td>35</td>
<td>70</td>
<td>35</td>
<td>85</td>
<td>50</td>
</tr>
<tr>
<td>7</td>
<td>40</td>
<td>80</td>
<td>40</td>
<td>75</td>
<td>45</td>
</tr>
<tr>
<td>8</td>
<td>30</td>
<td>80</td>
<td>40</td>
<td>75</td>
<td>50</td>
</tr>
<tr>
<td>9</td>
<td>45</td>
<td>75</td>
<td>45</td>
<td>75</td>
<td>50</td>
</tr>
<tr>
<td>10</td>
<td>55</td>
<td>75</td>
<td>40</td>
<td>85</td>
<td>35</td>
</tr>
</tbody>
</table>

**Solution:**

For each combination, consider the minimum score and the maximum score separately and calculate the average in each case.
Consider the mean values obtained for the minimum and maximum of each combination and calculate the range for each combination as

\[
\text{Range} = \text{Maximum Value} - \text{Minimum Value}
\]

The measure of importance for each combination is calculated as follows:

\[
\text{Measure of importance for a combination of attributes} = \frac{\text{Range for that combination}}{\text{Sum of the ranges for all the combinations}} \times 100
\]

Tabulate the results as follows:

<table>
<thead>
<tr>
<th>Combination</th>
<th>Max. Value</th>
<th>Min. Value</th>
<th>Range</th>
<th>Measure of Importance</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>73</td>
<td>38</td>
<td>35</td>
<td>21.875</td>
</tr>
<tr>
<td>II</td>
<td>80</td>
<td>42</td>
<td>38</td>
<td>23.750</td>
</tr>
<tr>
<td>III</td>
<td>78</td>
<td>51</td>
<td>27</td>
<td>16.875</td>
</tr>
<tr>
<td>IV</td>
<td>75</td>
<td>46</td>
<td>29</td>
<td>18.125</td>
</tr>
<tr>
<td>V</td>
<td>80</td>
<td>49</td>
<td>31</td>
<td>19.375</td>
</tr>
</tbody>
</table>

Sum of the ranges 160 100.000

Inference:

It is concluded that combination II is the most important one and combination III is the least important one.

**APPROACHES FOR CONJOINT ANALYSIS**

The following two approaches are available for conjoint analysis:

1. Multi-factor evaluation approach
2. Two-factor evaluation approach
MULTI-FACTOR EVALUATION APPROACH IN CONJOINT ANALYSIS

Suppose a researcher has to analyze n factors. It is possible that each factor can assume a value in different levels.

Product Profile

A product profile is a description of all the factors under consideration, with any one level for each factor.

Suppose, for example, there are 3 factors with the levels given below.

<table>
<thead>
<tr>
<th>Factor</th>
<th>Levels</th>
</tr>
</thead>
<tbody>
<tr>
<td>Factor 1</td>
<td>3 levels</td>
</tr>
<tr>
<td>Factor 2</td>
<td>2 levels</td>
</tr>
<tr>
<td>Factor 3</td>
<td>4 levels</td>
</tr>
</tbody>
</table>

Then we have \(3 \times 2 \times 4 = 24\) product profiles. For each respondent in the research survey, we have to provide 24 data sheets such that each data sheet contains a distinct profile. In each profile, the respondent is requested to indicate his preference for that profile in a rating scale of 0 to 10. A rating of 10 indicates that the respondent’s preference for that profile is the highest and a rating of 0 means that he is not all interested in the product with that profile.

Example:

consider the product ‘Refrigerator’ with the following factors and levels:

<table>
<thead>
<tr>
<th>Factor</th>
<th>Levels</th>
</tr>
</thead>
<tbody>
<tr>
<td>Factor 1</td>
<td>Capacity of 180 liters; 200 liters; 230 liters</td>
</tr>
<tr>
<td>Factor 2</td>
<td>Number of doors: either 1 or 2</td>
</tr>
<tr>
<td>Factor 3</td>
<td>Price: rs. 9000; rs. 10,000; rs. 12,000</td>
</tr>
</tbody>
</table>

Sample profile of the product

<table>
<thead>
<tr>
<th>Profile Number</th>
<th>Capacity</th>
<th>Number of Doors</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>200 liters</td>
<td>1</td>
</tr>
</tbody>
</table>
Price : Rs. 10,000
Rating of Respondent:
(in the scale of 0 to 10)

Steps In Multi-Factor Evaluation Approach:

1. Identify the factors or features of a product to be analyzed. If they are too many, select the important ones by discussion with experts.

2. Find out the levels for each factor selected in step 1.

3. Design all possible product profiles. If there are n factors with levels $L_1, L_2, \ldots, L_n$ respectively, then the total number of profiles = $L_1L_2\ldots L_n$.

4. Select the scaling technique to be adopted for multi-factor evaluation approach (rating scale or ranking method).

5. Select the list of respondents using the standard sampling technique.

6. Request each respondent to give his rating scale for all the profiles of the product. Another way of collecting the responses is to request each respondent to award ranks to all the profiles: i.e., rank 1 for the best profile, rank 2 for the next best profile etc.

7. For each factor profile, collect all the responses from all the participating respondents in the survey work.

8. With the rating scale awarded by the respondents, find out the score secured by each profile.

9. Tabulate the results in step 8. Select the profile with the highest score. This is the most preferred profile.

10. Implement the most preferred profile in the design of a new product.

Two-Factor Evaluation Approach In Conjoint Analysis

When several factors with different levels for each factor have to be analyzed, the respondents will have difficulty in evaluating all the profiles in the multi-factor evaluation approach. Because of this reason, two-factor evaluation approach is widely used in conjoint analysis.
Suppose there are several factors to be analyzed with different levels of values for each factor, then we consider any two factors at a time with their levels of values. For each such case, we have a data sheet called a **two-factor table**. If there are n factors, then the number of such data sheets is

\[
\binom{n}{2} = \frac{n(n-1)}{2}
\]

Let us consider the example of ‘Refrigerator’ described in the multi-factor approach. For the two factors (i) capacity and (ii) price, we have the following data sheet.

**Data Sheet (Two Factor Table) No:**

<table>
<thead>
<tr>
<th>Factor: Capacity of Refrigerator</th>
<th>Price</th>
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<tr>
<td></td>
<td>Rs. 9,000</td>
</tr>
<tr>
<td>180 liters</td>
<td></td>
</tr>
<tr>
<td>200 liters</td>
<td></td>
</tr>
<tr>
<td>230 liters</td>
<td></td>
</tr>
</tbody>
</table>

In this case, the data sheet is a matrix of 3 rows and 3 columns. Therefore, there are 3 × 3 = 9 places in the matrix. The respondent has to award ranks from 1 to 9 in the cells of the matrix. A rank of 1 means the respondent has the maximum preference for that entry and a rank of 9 means he has the least preference for that entry. Compared to multi-factor evaluation approach, the respondents will find it easy to respond to two-factor evaluation approach since only two factors are considered at a time.

**Steps in two-factor evaluation approach:**

1. Identify the factors or features of a product to be analyzed.

2. Find out the levels for each factor selected in step 1.

3. Consider all possible pairs of factors. If there are n factors, then the
number of pairs is \( \binom{n}{2} = \frac{n(n-1)}{2} \). For each pair of factors, prepare a two-factor table, indicating all the levels for the two factors. If \( L_1 \) and \( L_2 \) are the respective levels for the two factors, then the number of cells in the corresponding table is \( L_1L_2 \).

3. Select the list of respondents using the standard sampling technique.

4. Request each respondent to award ranks for the cells in each two-factor table. I.e., rank 1 for the best cell, rank 2 for the next best cell, etc.

5. For each two-factor table, collect all the responses from all the participating respondents in the survey work.

6. With the ranks awarded by the respondents, find out the score secured by each cell in each two-factor table.

7. Tabulate the results in step 7. Select the cell with the highest score. Identify the two factors and their corresponding levels.

8. Implement the most preferred combination of the factors and their levels in the design of a new product.

**Application:**

The two factor approach is useful when a manager goes for market segmentation to promote his product. The approach will enable the top level management to evolve a policy decision as to which segment of the market has to be concentrated more in order to maximize the profit from the product under consideration.

**QUESTIONS**

1. Explain the purpose of ‘factor analysis’.
2. What is the objective of ‘conjoint analysis’? Explain.
3. State the steps in the development of conjoint analysis.
4. State the applications of conjoint analysis.
5. Enumerate the advantages and disadvantages of conjoint analysis.
6. What is a ‘product profile’? Explain.
7. What are the steps in multi-factor evaluation approach in conjoint analysis?
8. What is a ‘two-factor table’? Explain.
9. Explain two-factor evaluation approach in conjoint analysis.

REFERENCES

**Statistical Table-1: F-values at 1% level of significance**

$df_1$: degrees of freedom for greater variance  
$df_2$: degrees of freedom for smaller variance

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**Statistical Table-2: F-values at 2.5% level of significance**

- $df_1$: degrees of freedom for greater variance
- $df_2$: degrees of freedom for smaller variance

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Statistical Table-3: F-values at 5% level of significance

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$df_j$: degrees of freedom for smaller variance

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### Statistical Table-4: F-values at 10% level of significance

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**df₂**: degrees of freedom for smaller variance

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<td>2.8</td>
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<td>2.2</td>
<td>2.1</td>
<td>2.0</td>
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UNIT V

1. Structure And Components Of Research Reports

Lesson Objectives:

- What Is A Report?
- Characteristics Of A Good Report
- Framework Of A Report
- Practical Reports Vs Academic Reports
- Parts Of A Research Report
- A Note On Literature Review

Learning Objectives:

After reading this lesson, you should be able to:

- Understand the meaning of a research report
- Analyze the components of a good report
- Structure of a report
- Characteristic differences in research reporting

WHAT IS A REPORT?

A report is a written document on a particular topic, which conveys information and ideas and may also make recommendations. Reports often form the basis of crucial decision making. Inaccurate, incomplete and poorly written reports fail to achieve their purpose and reflect on the decision, which will ultimately be made. This will also be the case if the report is excessively long, jargonistic and/ or structureless. A good report can be written by keeping the following features in mind:

1. All points in the report should be clear to the intended reader.
2. The report should be concise with information kept to a necessary
minimum and arranged logically under various headings and sub-headings.

3. All information should be correct and supported by evidence.

4. All relevant material should be included in a complete report.

**Purpose Of Research Report**

1. Why am I writing this report? Do I want to inform/ explain/ persuade, or indeed all of these.

2. Who is going to read this report? Managers/ academicians/ researchers! What do they already know? What do they need to know? Do any of them have certain attitudes or prejudices?

3. What resources do we have? Do I have access to a computer? Do I have enough time? Can any of my colleagues help?

4. Think about the content of your report – what am I going to put in it? What are my main themes? How much should be the text, and how much should be the illustrations?

**Framework Of A Report**

The various frameworks can be used depending on the content of the report, but generally the same rules apply. Introduction, method, results and discussion with references or bibliography at the end, and an abstract at the beginning could form the framework.

**STRUCTURE OF A REPORT**

Structure your writing around the IMR&D framework and you will ensure a beginning, middle and end to your report.

<table>
<thead>
<tr>
<th>I</th>
<th>Introduction</th>
<th>Why did I do this research?</th>
<th>(beginning)</th>
</tr>
</thead>
<tbody>
<tr>
<td>M</td>
<td>Method</td>
<td>What did I do and how did I go about doing it?</td>
<td>(middle)</td>
</tr>
<tr>
<td>R</td>
<td>Results</td>
<td>What did I find?</td>
<td>(middle)</td>
</tr>
</tbody>
</table>

AND
What do I put in the beginning part?

<table>
<thead>
<tr>
<th><strong>TITLE PAGE</strong></th>
<th>Title of project, Sub–title (where appropriate), Date, Author, Organization, Logo</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>BACKGROUND</strong></td>
<td>History(if any) behind project</td>
</tr>
<tr>
<td><strong>ACKNOWLEDGEMENT</strong></td>
<td>Author thanks people and organization who helped during the project</td>
</tr>
<tr>
<td><strong>SUMMARY</strong>(sometimes called abstract of the synopsis)</td>
<td>A condensed version of a report – outlines salient points, emphasizes main conclusions and (where appropriate) the main recommendations. N.B this is often difficult to write and it is suggested that you write it last.</td>
</tr>
<tr>
<td><strong>LIST OF CONTENTS</strong></td>
<td>An at- a – glance list that tells the reader what is in the report and what page number(s) to find it on.</td>
</tr>
<tr>
<td><strong>LIST OF TABLES</strong></td>
<td>As above, specifically for tables.</td>
</tr>
<tr>
<td><strong>LIST OF APPENDICES</strong></td>
<td>As above, specifically for appendices.</td>
</tr>
<tr>
<td><strong>INTRODUCTION</strong></td>
<td>Author sets the scene and states his/ her intentions.</td>
</tr>
<tr>
<td><strong>AIMS AND OBJECTIVES</strong></td>
<td>AIMS – general aims of the audit/ project, broad statement of intent. OBJECTIVES – specific things expected to do/ deliver(e.g. expected outcomes)</td>
</tr>
</tbody>
</table>
### What do I Put In the middle Part?

<table>
<thead>
<tr>
<th><strong>METHOD</strong></th>
<th>Work steps; what was done – how, by whom, when?</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>RESULT/FINDINGS</strong></td>
<td>Honest presentation of the findings, whether these were as expected or not. Give the facts, including any inconsistencies or difficulties encountered</td>
</tr>
</tbody>
</table>

### What do I put in the end part?

| **DISCUSSION** | Explanation of the results. (you might like to keep the SWOT analysis in mind and think about your project's strengths, weakness, opportunities and threats, as you write) |
| **CONCLUSIONS** | The author links the results/findings with the points made in the introduction and strives to reach clear, simply stated and unbiased conclusions. Make sure they are fully supported by evidence and arguments of the main body of your audit/project. |
| **RECOMMENDATIONS** | The author states what specific actions should be taken, by whom and why. They must always be linked to the future and should always be realistic. Don't make them unless asked to. |
| **REFERENCES** | A section of a report, which provides full details of publications mentioned in the text, or from which extracts have been quoted. |
APPENDIX

The purpose of an appendix is to supplement the information contained in the main body of the report.

PRACTICAL REPORTS VS. ACADEMIC REPORTS

Practical Reports:

In the practical world of business or government, a report conveys an information and (sometimes) recommendations from a researcher who has investigated a topic in detail. A report like this will usually be requested by people who need the information for a specific purpose and their request may be written in terms of reference or the brief. Whatever the report, it is important to look at the instruction for what is wanted. A report like this differs from an essay in that it is designed to provide information which will be acted on, rather than to be read by people interested in the ideas for their own sake. Because of this, it has a different structure and layout.

Academic Reports:

A report written for an academic course can be thought of as a simulation. We can imagine that someone wants the report for a practical purpose, although we are really writing the report as an academic exercise for assessment. Theoretical ideas will be more to the front in an academic report than in a practical one. Sometimes a report seems to serve academic and practical purposes. Students on placement with organizations often have to produce a report for the organization and for assessment on the course. Although the background work for both will be related, in practice, the report the student produces for academic assessment will be different from the report produced for the organization, because the needs of each are different.
RESEARCH REPORT: PRELIMINARIES

It is not sensible to leave all your writing until the end. There is always the possibility that it will take much longer than you anticipate and you will not have enough time. There could also be pressure upon available word processors as other students try to complete their own reports. It is wise to begin writing up some aspects of your research as you go along. Remember that you do not have to write your report in the order it will be read. Often it is easiest to start with the method section. Leave the introduction and the abstract to last. The use of a word processor makes it very straightforward to modify and rearrange what you have written as your research progresses and your ideas change. The very process of writing will help your ideas to develop. Last but by no means least, ask someone to proofread your work.

STRUCTURE OF A RESEARCH REPORT

A research report has a different structure and layout in comparison to a project report. A research report is for reference and is often quite a long document. It has to be clearly structured for the readers to quickly find the information wanted. It needs to be planned carefully to make sure that the information given in the report is put under correct headings.

PARTS OF RESEARCH REPORT

Cover sheet: This should contain some or all of the following:
Full title of the report
Name of the researcher
Name of the unit of which the project is a part
Name of the institution
Date/year.

Title page: Full title of the report.
Your name
Acknowledgement: a thanks giving to the people who helped you.
Contents
List of the Tables

Headings and sub-headings used in the report should be given with their page numbers. Each chapter should begin on a new page. Use a consistent system in dividing the report into parts. The simplest may be to use chapters for each major part and subdivide these into sections and sub-sections. 1, 2, 3 etc. Can be used as the numbers for each chapter. The sections of chapter 3 (for example) would be 3.1, 3.2, 3.3, and so on. For further sub-division of a sub-section you may use 3.2.1, 3.2.2, and so on.

Abstract or Summary or Executive Summary or Introduction:

This presents an overview of the whole report. It should let the reader see in advance, what is in the report. This includes what you set out to do, how review of literature is focused and narrowed in your research, the relation of the methodology you chose to your objectives, a summary of your findings and analysis of the findings.

BODY

Aims And Purpose or Aims And Objectives:

Why did you do this work? What was the problem you were investigating? If you are not including review of literature, mention the specific research/es which is/are relevant to your work.

Review of Literature

This should help to put your research into a background context and to explain its importance. Include only the books and articles which relate directly to your topic. You need to be analytical and critical, and not just describe the works that you have read.

Methodology

Methodology deals with the methods and principles used in an activity, in this case research. In the methodology chapter, explain the method/s you used for the research and why you thought they were the appropriate
ones. You may, for example, be depending mostly upon secondary data or you might have collected your own data. You should explain the method of data collection, materials used, subjects interviewed, or places you visited. Give a detailed account of how and when you carried out your research and explain why you used the particular method/s, rather than other methods. Included in this chapter should be an examination of ethical issues, if any.

Results or Findings

What did you find out? Give a clear presentation of your results. Show the essential data and calculations here. You may use tables, graphs and figures.

Analysis and Discussion

Interpret your results. What do you make out of them? How do they compare with those of others who have done research in this area? The accuracy of your measurements/results should be discussed and deficiencies, if any, in the research design should be mentioned.

Conclusions

What do you conclude? Summarize briefly the main conclusions which you discussed under “Results.” were you able to answer some or all of the questions which you raised in your aims and objectives? Do not be tempted to draw conclusions which are not backed up by your evidence. Note the deviation/s from expected results and any failure to achieve all that you had hoped.

Recommendations

Make your recommendations, if required. The suggestions for action and further research should be given.

Appendix

You may not need an appendix, or you may need several. If you have used questionnaires, it is usual to include a blank copy in the appendix. You could include data or calculations, not given in the body, that are
necessary, or useful, to get the full benefit from your report. There may be maps, drawings, photographs or plans that you want to include. If you have used special equipment, you may include information about it.

The plural of an appendix is appendices. If an appendix or appendices are needed, design them thoughtfully in a way that your readers find it/them convenient to use.

References

List all the sources which you referred in the body of the report. You may use the pattern prescribed by American Psychological Association, or any other standard pattern recognized internationally.

REVIEW OF LITERATURE

In the case of small projects, this may not be in the form of a critical review of the literature, but this is often asked for and is a standard part of larger projects. Sometimes students are asked to write Review of Literature on a topic as a piece of work in its own right. In its simplest form, the review of literature is a list of relevant books and other sources, each followed by a description and comment on its relevance.

The literature review should demonstrate that you have read and analysed the literature relevant to your topic. From your readings, you may get ideas about methods of data collection and analysis. If the review is part of a project, you will be required to relate your readings to the issues in the project, and while describing the readings, you should apply them to your topic. A review should include only relevant studies. The review should provide the reader with a picture of the state of knowledge in the subject.

Your literature search should establish what previous researches have been carried out in the subject area. Broadly speaking, there are three kinds of sources that you should consult:

1. Introductory material;
2. Journal articles and

To get an idea about the background of your topic, you may consult
one or more textbooks at the appropriate time. It is a good practice to review in cumulative stages - that is, do not think you can do it all at one go. Keep a careful record of what you have searched, how you have gone about it, and the exact citations and page numbers of your readings. Write notes as you go along. Record suitable notes on everything you read and note the methods of investigations. Make sure that you keep a full reference, complete with page numbers. You will have to find your own balance between taking notes that are too long and detailed, and ones too brief to be of any use. It is best to write your notes in complete sentences and paragraphs, because research has shown that you are more likely to understand your notes later if they are written in a way that other people would understand. Keep your notes from different sources and/or about different points on separate index cards or on separate sheets of paper.

You will do mainly basic reading while you are trying to decide on your topic. You may scan and make notes on the abstracts or summaries of work in the area. Then do a more thorough job of reading later on, when you are more confident of what you are doing. If your project spans several months, it would be advisable towards the end to check whether there are any new and recent references.

REFERENCES

There are many methods of referencing your work; some of the most common ones are the numbered style, american psychological association style and the harvard method, with many other variations. Just use the one you are most familiar and comfortable with. Details of all the works referred by you should be given in the reference section.

THE PRESENTATION OF REPORT

Well-produced, appropriate illustrations enhance the presentability of a report. With today's computer packages, almost anything is possible. However, histograms, bar charts and pie charts are still the three 'staples'. Readers like illustrated information, because it is easier to absorb and it's more memorable. Illustrations are useful only when they are easier to understand than words or figures and they must be relevant to the text. Use the algorithm included to help you decide whether or not to use an illustration. They should never be included for their own sake, and don't overdo it; too many illustrations distract the attention of readers.
UNIT V

2. Types Of Reports: Characteristics Of Good Research Report

Lesson Outline:

- Different Types Of Reports
- Technical Reports
- General Reports
- Reporting Styles
- Characteristics Of A Good Report

Learning Objectives:

After reading this lesson, you should be able to:

- Understand different types of reports
- Technical reports and their contents
- General reports
- Different types of writing styles
- Essential characteristics of a good report

Reports vary in length and type. Students’ study reports are often called term papers, project reports, theses, dissertations depending on the nature of the report. Reports of researchers are in the form of monographs, research papers, research thesis, etc. In business organizations a wide variety of reports are under use: project reports, annual reports of financial statements, report of consulting groups, project proposals etc. News items in daily papers are also one form of report writing. In this lesson, let us identify different forms of reports and their major components.
Types of reports

Reports may be categorized broadly as Technical Reports and General Reports based on the nature of methods, terms of reference and the extent of in-depth enquiry made etc. On the basis of usage pattern, the reports may also be classified as information oriented reports, decision oriented reports and research based reports. Further, reports may also differ based on the communication situation. For example, the reports may be in the form of Memo, which is appropriate for informal situations or for short periods. On the other hand, the projects that extend over a period of time, often call for project reports. Thus, there is no standard format of reports. The most important thing that helps in classifying the reports is the outline of its purpose and answers for the following questions:

- What did you do?
- Why did you choose the particular research method that you used?
- What did you learn and what are the implications of what you learned?
- If you are writing a recommendation report, what action are you recommending in response to what you learned?

Two types of report formats are described below:

A Technical Report

A technical report mainly focuses on methods employed, assumptions made while conducting a study, detailed presentation of findings and drawing inferences and comparisons with earlier findings based on the type of data drawn from the empirical work.

An outline of a Technical Report mostly consists of the following:

Title and nature of the study:

- Brief title and the nature of work sometimes followed by subtitle indicate more appropriately either the method or tools used. Description of objectives of the study, research design, operational terms, working hypothesis, type of analysis and data required should be present.
Abstract of Findings:

A brief review of the main findings just can be made either in a paragraph or in one/two pages.

Review of current status:

A quick review of past observations and contradictions reported, applications observed and reported are reviewed based on the in-house resources or based on published observations.

Sampling and Methods employed

Specific methods used in the study and their limitations. In the case of experimental methods, the nature of subjects and control conditions are to be specified. In the case of sample studies, details of the sample design i.e., sample size, sample selection etc are given.

Data sources and experiment conducted

Sources of data, their characteristics and limitations should be specified. In the case of primary survey, the manner in which data has been collected should be described.

Analysis of data and tools used.

The analysis of data and presentation of findings of the study with supporting data in the form of tables and charts are to be narrated. This constitutes the major component of the research report.

Summary of findings

A detailed summary of findings of the study and major observations should be stated. Decision inputs if any, policy implications from the observations should be specified.

References

A brief list of studies conducted on similar lines, either preceding the present study or conducted under different experimental conditions is listed.
Technical appendices

These appendices include the design of experiments or questionnaires used in conducting the study, mathematical derivations, elaboration on particular techniques of analysis etc.

General Reports

General reports often relate popular policy issues mostly related to social issues. These reports are generally simple, less technical, good use of tables and charts. Most often they reflect the journalistic style. Example for this type of report is the “Best B-Schools Survey in Business Magazines”. The outline of these reports is as follows:

1. Major Findings and their Implications
2. Recommendations for Action
3. Objectives of the Study
4. Method Employed for Collecting Data
5. Results

Writing Styles

There are atleast 3 distinct report writing styles that can be applied by students of Business Studies. They are called:

1. Conservative
2. Key points
3. Holistic

1. Conservative Style

Essentially, the conservative approach takes the best structural elements from essay writing and integrates these with appropriate report writing tools. Thus, headings are used to deliberate upon different sections of the answer. In addition, the space is well utilized by ensuring that each paragraph is distinct (perhaps separated from other paragraphs by leaving two blank lines in between).
2. **Key Point Style**

This style utilizes all of the report writing tools and is thus more overtly 'report-looking'. Use of headings, underlining, margins, diagrams and tables are common. Occasionally reporting might even use indentation and dot points. The important thing to remember is that the tools should be applied in a way that adds to the report. The question must be addressed and the tools applied should assist in doing that. An advantage of this style is the enormous amount of information that can be delivered relatively quickly.

3. **Holistic Style**

The most complex and unusual of the styles, holistic report writing aims to answer the question from a thematic and integrative perspective. This style of report writing requires the researcher to have a strong understanding of the course and is able to see which outcomes are being targeted by the question.

**Essentials Of A Good Report:**

Good research report should satisfy some of the following basic characteristics:

**STYLE**

Reports should be easy to read and understand. The style of the writer should ensure that sentences are succinct and the language used is simple, to the point and avoiding excessive jargon.

**LAYOUT**

A good layout enables the reader to follow the report’s intentions, and aids the communication process. Sections and paragraphs should be given headings and sub¬-headings. You may also consider a system of numbering or lettering to identify the relative importance of paragraphs and sub-paragraphs. Bullet points are an option for highlighting important points in your report.
ACCURACY

Make sure everything you write is factually accurate. If you would mislead or misinform, you will be doing a disservice not only to yourself but also to the readers, and your credibility will be destroyed. Remember to refer to any information you have used to support your work.

CLARITY

Take a break from writing. When you would come back to it, you’ll have the degree of objectivity that you need. Use simple language to express your point of view.

READABILITY

Experts agree that the factors, which affect readability the most, are:

- Attractive appearance
- Non-technical subject matter
- Clear and direct style
- Short sentences
- Short and familiar words

REVISION

When first draft of the report is completed, it should be put to one side at least for 24 hours. The report should then be read as if with eyes of the intended reader. It should be checked for spelling and grammatical errors. Remember the spell and grammar check on your computer. Use it!

REINFORCEMENT

Reinforcement usually gets the message across. This old adage is well known and is used to good effect in all sorts of circumstances e.g., presentations - not just report writing.

- TELL THEM WHAT YOU ARE GOING TO SAY: in the introduction and summary you set the scene for what follows in your report.
THEN SAY IT: you spell things out in results/findings

THEN TELL THEM WHAT YOU SAID: you remind your readers through the discussion what it was all about.

FEEDBACK MEETING

It is useful to circulate copies of your report prior to the feedback meeting. Meaningful discussion can then take place during the feedback meeting with recommendations for change more likely to be agreed upon which can then be included in your conclusion. The following questions should be asked at this stage to check whether the Report served the purpose:

- Does the report have impact?
- Do the summary/abstract do justice to the report?
- Does the introduction encourage the reader to read more?
- Is the content consistent with the purpose of the report?
- Have the objectives been met?
- Is the structure logical and clear?
- Have the conclusions been clearly stated?
- Are the recommendations based on the conclusions and expressed clearly and logically?

***
UNIT V

3. Format And Presentation Of A Report

Lesson Outline:

» Importance Of Presentation Of A Report
» Common Elements Of A Format
» Title Page
» Introductory Pages
» Body Of The Text
» References
» Appendix
» Dos And Don'ts
» Presentation Of Reports

Learning Objectives:

After reading this lesson, you should be able to:

» Understand the importance of format of a report
» Contents of a title page
» What should be in introductory pages
» Contents of a body text
» How to report other studies
» Contents of an appendix
» Dos and don'ts of a report

Any report serves its purpose, if it is finally presented before the stakeholders of the work. In the case of an MBA student, Project Work undertaken in an industrial enterprise and the findings of the study would be more relevant, if they are presented before the internal managers of the company. In the case of reports prepared out of consultancy projects,
a presentation would help the users to interact with the research team and get clarification on any issue of their interest. Business Reports or Feasibility Reports do need a summary presentation, if they have to serve the intended purpose. Finally, the Research Reports of the scholars would help in achieving the intended academic purpose, if they are made public in academic symposiums, seminars or in Public Viva Voce examinations. Thus, the presentation of a report goes along with preparation of a good report. Further, the use of graphs, charts, citations and pictures draw the attention of readers and audience of any type. In this lesson, it is intended to provide a general outline related to the presentation of any type of report. See Exhibit I

**Exhibit I**

**Common Elements Of A Report**

A report may contain some or all of the following, please refer to your departmental guidelines.

**MEMORANDUM OR COVERING LETTER**

Memorandum or Covering Letter is a brief note stating the purpose or giving an explanation that is used when the report is sent to someone within the same organization.

**TITLE PAGE**

It is addressed to the receiver of a report while giving an explanation for it, and is used when the report is for someone who does not belong to the same organization as the writer. It contains a descriptive heading or name. It may also contain author’s name, position, company’s name and so on.

**EXECUTIVE SUMMARY**

Executive summary summarizes the main contents and is usually of about 300-350 words.
TABLE OF CONTENTS

Table of Contents consists of a list of the main sections, indicating the page on which each section begins.

INTRODUCTION

Informs the reader of what the report is about—aim and purpose, significant issues, any relevant background information.

REVIEW OF LITERATURE

Presents critical analysis of the available research to build a base for the present study.

METHODOLOGY

Gives details about nature of the study, research design, sample, and tools used for data collection and analysis.

RESULTS

Presents findings of the study.

DISCUSSION

Describes the reasoning and research in detail.

CONCLUSION/S

Summarizes the main points made in the written work in the light of objectives. It often includes an overall answer to the problem/s addressed; or an overall statement synthesizing the strands of information dealt with.

RECOMMENDATION/S OR IMPLICATIONS

Gives suggestions related to the issue(s) or problem(s) dealt with. It may highlight the applications of the findings under implications section.
REFERENCES

An alphabetical list of all sources referred in the report.

APPENDICES

Extra information of further details placed after the main body of the text.

FORMATS OF REPORTS

Before attempting to look into presentation dimensions of a Report, a quick look into standard format associated with a Research Report is examined hereunder. The format generally includes the steps one should follow while writing and finalizing their research report.

Different Parts of a Report

Generally different parts of a report include:

1. Cover Page / Title Page

2. Introductory Pages (Foreword, Preface, Acknowledgement, Table of Contents, List of Tables, List of Illustrations or Figures, Key Words / Abbreviations Used Etc.)

3. Contents of the Report (Which Generally Includes a Macro Setting, Research Problem, Methodology Used, Objectives of the Study, Review of Studies, Tools Used for Data Collection and Analysis, Empirical Results in One/Two Sections, Summary of Observations etc.)

4. References (Including Appendices, Glossary of Terms Used, Source Data, Derivations of Formulas for Models Used in the Analysis etc.)
Title Page:

The Cover Page or Title Page of a Research Report should contain the following information:

1. Title of the Project / Subject
2. Who has conducted the study
3. For what purpose
4. Organization
5. Period of submission

***
A Model:

An example of a Summer Project Report conducted by an MBA student generally follows the following Title Page:

A STUDY ON THE USE OF COMPUTER TECHNOLOGY IN BANKING OPERATIONS IN XXX BANK LTD., PONDICHERRY

A SUMMER PROJECT REPORT
PREPARED BY
Ms. MADAVI LATHA

Submitted at

SCHOOL OF MANAGEMENT
PONDICHERRY UNIVERSITY
PONDICHERRY – 605 014
2006

Introductory Pages:

Introductory pages generally do not constitute the Write up of the Research work done. These introductory pages basically form the Index of the work done. These pages are usually numbered in Roman numerical (eg, I, ii, iii etc). The introductory pages include the following components
Foreword is usually one page write up or a citation about the work by any eminent / popular personality or a specialist in the given field of study. Generally, the write up includes a brief background on the contemporary issues and suitability of the present subject and its timeliness, major highlights of the present work, brief background of the author etc. The writer of the foreword generally gives the foreword on his letter head.

Preface is again one/two pages write up by the author of the book / report stating circumstances under which the present work is taken up, importance of the work, major dimensions examined and intended audience for the given work. The author gives his signature and address at the bottom of the page along with date and year of the work.

Acknowledgements is a short section, mostly a paragraph. It mostly consists of sentences giving thanks to all those associated and encouraged to carry out the present work. Generally, author takes time to acknowledge the liberal funding by any funding agency to carry out the work, and agencies which had given permission to use their resources. At the end, the author thanks everybody and gives his signature.

Table of Contents refers to the index of all pages of the said Research Report. These contents provide the information about the chapters, subsections, annexure for each chapter, if any, etc. Further, the page numbers of the content of the report greatly helps any one to refer to those pages for necessary details. Most authors use different forms while listing the sub contents. These include alphabet classification and decimal classification.

Examples for both of them are given below:
Example of content sheet (alphabet classification)
An example of Content Sheet with decimal classification

240
CONTENTS

Foreword i

Preface iii

Acknowledgement v

Chapter I (Title of the Chapter) INTRODUCTION

1. Macro economic background 1

2. Performance of a specific industry sector 6

3. Different studies conducted so far 9

4 nature and scope 17

4.1. Objectives of the study 18

4.2. Methodology adopted 19

4.2. A. Sampling Procedure adopted 20

4.2.b. Year of the study 20

Chapter II (Title of the Chapter): Empirical Results I 22

1. Test Results of H1 22

2. Test Results of H2 27

3 test Results of H3 32

3.1. Sub Hypothesis of H3 33
List of Tables and Charts:

Details of Charts and Tables given in the Research Report are numbered and presented on separate pages and the lists of such tables and charts are given on a separate page. Tables are generally numbered either in Arabic numerals or in decimal form. In the case of decimal form, it is possible to indicate the chapter to which the said table belongs. For example, Table 2.1 refers to Table 1 in Chapter 2.

Executive Summary:

Most Business Reports or Project works conducted on a specific issue carry one or two pages of Executive Summary. This summary precedes the chapters of the Regular Research Report. This summary generally contains a brief description of problem under enquiry, methods used and the findings. A line about the possible alternatives for decision making would be the last line of the Executive Summary.

BODY OF THE REPORT:

The body of the Report is the most important part of the report. This body of report may be segmented into a handful of Units or Chapters
arranged in a sequential order. Research Report often present the Methodology, Objectives of the study, data tools, etc in the first or second chapters along with a brief background of the study, review of relevant studies. The major findings of the study are incorporated into two or three chapters based on the major or minor hypothesis tested or based on the sequence of objectives of the study. Further, the chapter plan may also be based likely on different dimensions of the problem under enquiry.

Each Chapter may be divided into sections. While the first section may narrate the descriptive characteristics of the problem under enquiry, the second and subsequent sections may focus on empirical results based on deeper insights of the problem of study. Each chapter based on research studies mostly contain major headings, sub headings, quotations drawn from observations made by earlier writers, footnotes and exhibits.

Use of References:

There are two types of reference formatting. The first is the ‘in-text’ reference format, where previous researchers and authors are cited during the building of arguments in the introduction and discussion sections. The second type of format is that adopted for the Reference section for writing footnotes or Bibliography.

Citations in the text

The names and dates of researchers go in the text as they are mentioned e.g., “This idea has been explored in the work of Smith (1992).” it is generally unacceptable to refer to authors and previous researchers etc.

Examples of Citing References (Single Author)

Duranti (1995) has argued or it has been argued that (Duranti, 1995)

In the case of more authors,

Moore, Maguire, and Smyth (1992) proposed or it has been proposed that (Moore, Macquire, & Smyth, 1992)
For subsequent citations in the same report: Moore et al. (1992) also proposed... Or it has also been proposed that.... (Moore et al., 1992)

The reference section:

The report ends with reference section, which comes immediately after the Recommendations and begins on a new page. It is titled as 'References' in upper and lower case letters centered across the page.

Published Journal Articles


Authored Books


APPENDICES:

The purpose of the appendices is to supplement the main body of your text and provide additional information that may be of interest to the reader.

There is no major heading for the appendices. You simply need to include each one, starting on a new page, numbered, using capital letters, and headed with a centered brief descriptive title. For example:

Appendix A: List of stimulus words presented to the participants
Dos and Don’ts of Report Writing

1. Choose a font size that is not too small or too large; 11 or 12 is a good font size to use.

2. Acknowledgment need not be a separate page, except in the final report. In fact, you could just drop it altogether for the first- and second-stage reports. Your guide already knows how much you appreciate his/her support. Express your gratitude by working harder instead of writing a flowery acknowledgment.

3. Make sure your paragraphs have some indentation and that it is not too large. Refer to some text books or journal papers if you are not sure.

4. If figures, equations, or trends are taken from some reference, the reference must be cited right there, even if you have cited it earlier.

5. The correct way of referring to a figure is Fig. 4 or Fig. 1.2 (note that there is a space after Fig.). The same applies to Section, Equation, etc. (e.g., sec. 2, eq. 3.1).

6. Cite a reference as, for example, “The threshold voltage is a strong function of the implant dose [1].” note that there must be a space before the bracket.

7. Follow some standard format while writing references. For example, you could look up any IEEE transactions issue and check out the format for journal papers, books, conference papers, etc.

8. Do not type references (for that matter, any titles or captions) entirely in capital letters. The only capital letters required are (i) the first letter of a name, (ii) acronyms, (iii) the first letter of the title of an article (iv) the first letter of a sentence.

9. The order of references is very important. In the list of your references, the first reference must be the one which is cited before any other reference, and so on. Also, every reference in the list must be cited at least once (this also applies to figures). In handling references and figure numbers, Latex turns out to be far better than Word.

10. Many commercial packages allow “screen dump” of figures. While this is useful in preparing reports, it is often very wasteful (in terms of toner or ink) since the background is black. Please see if you can invert
the image or use a plotting program with the raw data such that the background is white.

11. The following tips may be useful: (a) for windows, open the file in Paint and select Image/Invert Colors. (b) For Linux, open the file in Image Magick (this can be done by typing display) and then selecting Enhance/Negate.

12. As far as possible, place each figure close to the part of the text where it is referred to.

13. A list of figures is not required except for the final project report. It generally does not do more than wasting paper.

14. The figures, when viewed together with the caption, must be, as far as possible, self-explanatory. There are times when one must say, “see text for details”. However, this is an exception and not a rule.

15. The purpose of a figure caption is simply to state what is being presented in the figure. It is not the right place for making comments or comparisons; that should appear only in the text.

16. If you are showing comparison of two (or more) quantities, use the same notation throughout the report. For example, suppose you want to compare measured data with analytical model in four different figures, in each figure, make sure that the measured data is represented by the same line type or symbol. The same should be followed for the analytical model. This makes it easier for the reader to focus on the important aspects of the report rather than getting lost in lines and symbols.

17. If you must resize a plot or a figure, make sure that you do it simultaneously in both x and y directions. Otherwise, circles in the original figure will appear as ellipses, letters will appear too fat or too narrow, and other similar calamities will occur.

18. In the beginning of any chapter, you need to add a brief introduction and then start sections. The same is true about sections and subsections. If you have sections that are too small, it only means that there is not enough material to make a separate section. In that case, do not make a separate section. Include the same material in the main section or elsewhere.
19. Remember, a short report is perfectly acceptable if you have put in the effort and covered all important aspects of your work. Adding unnecessary sections and subsections will create the impression that you are only covering up the lack of effort.

20. Do not make one-line paragraphs.

21. Always add a space after a full stop, comma, colon, etc. Also, leave a space before opening a bracket. If the sentence ends with a closing bracket, add the full stop (or comma or semicolon, etc) after the bracket.

22. Do not add a space before a full stop, comma, colon, etc.

23. Using a hyphen can be tricky. If two (or more) words form a single adjective, a hyphen is required; otherwise, it should not be used. For example, (a) A short-channel device shows a finite output conductance. (b) This is a good example of mixed-signal simulation. (c) Several devices with short channels were studied.

24. If you are using Latex, do not use the quotation marks to open. If you do that, you get “this”. Use the single opening quotes (twice) to get “this”.

25. Do not use very informal language. Instead of “this theory should be taken with a pinch of salt,” you might say, “this theory is not convincing,” or “it needs more work to show that this theory applies in all cases.”

26. Do not use “&”; write “and” instead. Do not write “There’re” for “There are” etc.

27. If you are describing several items of the same type (e.g., short-channel effects in a MOS transistor), use the “list” option; it enhances the clarity of your report.

28. Do not use “bullets” in your report. They are acceptable in a presentation, but not in a formal report. You may use numerals or letters instead.

29. Whenever in doubt, look up a text book or a journal paper to verify whether your grammar and punctuation are correct.

30. Do a spell check before you print out your document. It always helps.

31. Always write the report so that the reader can easily make out what
your contribution is. Do not leave the reader guessing in this respect.

32. Above all, be clear. Your report must have a flow, i.e., the reader must be able to appreciate continuity in the report. After the first reading, the reader should be able to understand (a) the overall theme and (b) what is new (if it is a project report).

33. Plagiarism is a very serious offense. You simply cannot copy material from an existing report or paper and put it verbatim in your report. The idea of writing a report is to convey in your words what you have understood from the literature.

The above list may seem a little intimidating. However, if you make a sincere effort, most of the points are easy to remember and practice. A supplementary exercise that will help you immensely is that of looking for all major and minor details when you read an article from a newspaper or a magazine, such as grammar, punctuation, organization of the material, etc.

PRESENTATION OF A REPORT

In this section, we will look into the issues associated with presentation of a Research Report by the Researcher or Principal Investigator. While preparing for the presentation of a report, the researchers should focus on the following issues:

1. What is the purpose of the report and issues on which the Presentation has to focus?

2. Who are the stakeholders and what are their areas of interest?

3. The mode and media of presentation.

4. Extent of Coverage and depth to address at.

5. Time, Place and cost associated with presentation.

6. Audio – Visual aids intended to be used.
Case Study

1. Satisfying Customer Need:

A customer is approaching a Godrej Refrigerator (Exclusively) dealer. By considering the following how will you match the product to the customer for the satisfaction?

a. Vegetarian/Non Vegetarian
b. Non Vegetarian/Fost Free
c. Size of Refrigerator and Family
d. Brand first/Shop First
e. Buying Behavior/Changing Behaviour

2. A leading software services company has lived to as a researcher. The task before you is to assess the need for software products that will be accepted readily in the market. The company has not looked at software products and has no idea of the industry demand. Prepare a research proposal to identify the industrial segment and the type of software products that the company can consider manufacturing. The proposal should include the research design, time line and limitations.

3. Given below are the figures of production (in lakh kg) of a sugar factory.

<table>
<thead>
<tr>
<th></th>
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<th></th>
<th></th>
<th></th>
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<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Production</td>
<td>40</td>
<td>45</td>
<td>46</td>
<td>42</td>
<td>47</td>
<td>50</td>
<td>46</td>
</tr>
</tbody>
</table>

Fit a straight-line trend by the least squares method and tabulate the trend.

4. Genetic theory states that children having one parent of blood type A and the other of blood type B will always be of one of three types, A, AB, B and the proportion of three types will on an average be as 1 : 2 : 1. A report states that out of 300 children having one A parent and B parent, 30 per cent were found to be types A, 45 per cent type AB and remainder type B. Test the hypothesis by Chi square test.